

PRACTICAL MATHEMATICS

ISSUE
No.16

A Q U A R T E R L Y

WITH APPLICATIONS IN THE
FIELD OF BUSINESS

MATHEMATICS OF ACCOUNTING

Computations and Formulae
Accounting Statements
and Equations
Determination and Valuation
of Income
Rule of Debit and Credit
Analysis and Interpretation
Pre-Requisites
Bases Methods
Specialized Accounting Problems
Profit-and-Loss Analysis
Executives' Consolidated
Bonuses Statements
Interdependent State
and Federal Taxes
—ALSO—
Self-Tests and Mathematics Problems

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EDITOR: REGINALD STEVENS KIMBALL ED.D.

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CHATS WITH THE EDITOR

AT SOME time or other, almost everyone has occasion to "keep books", even if he does most of his personal accounting in the form of checkbook stubs. Any business, large or small, is called upon these days to furnish figures to federal and State officials in connection with the collection of income taxes and other revenue items. A complete knowledge of accounting procedures is the subject of many years of close study, leading to recognition as Certified Public Accountant.

In this issue of **PRACTICAL MATHEMATICS**, it is not our purpose to attempt to provide a complete course in accounting, but rather to set forth the dependence of accounting upon mathematical formulae and procedures. As a certain amount of accounting theory is necessary in order to understand the precise way in which these formulae are called into play, however, we do discuss some of the theoretical bases for the use of formulae.

In actual practice, a great deal of bookkeeping may be performed with only a knowledge of the fundamental operations — addition, subtraction, multiplication, and division. It is, as the reader will soon discover, a matter of "simple arithmetic". Confining one's efforts to these simple processes, nevertheless, often entails prolonged computations which may be shortened by the use of various tables and devices which take the heart-ache and the chance of human fallibility out of the operation.

Several times in the earlier issues of **PRACTICAL MATHEMATICS**, we have

stressed the importance of careful arrangement of the computations. Nowhere is this more important than in the field of accounting, where careful attention to the placing of figures in well-ordered columns, with proper spacing, is vital to the carrying through of a problem to its completion. The student will do well to acquire regular bookkeeping paper, ruled for ledger, journal, etc., like the samples which are printed in connection with solutions to various illustrative examples and problems. Failing this, it is worth the time spent to rule up the forms before beginning to set down the work. When using the typewriter for presenting the final figures, as in other examples shown, one should use care in spacing the columns properly and in making the proper indentations to indicate the relative importance of the items.

As shown in the early part of Dr. Wright's article, a review of the information contained in Issue Number Two of **PRACTICAL MATHEMATICS**, especially the section on ratios and percentages (pp. 72-77, and 83-87) will be a great help toward understanding the procedures in accounting. At various times when complicated multiplications and divisions are demanded, the reader who has really mastered the ability to use the slide rule (pp. 101-114) and the logarithm tables (pp. 88-100, 126, 127) will find that his time has not been wasted. Here, perhaps as much as anywhere else in the applied-mathematics field, he will have occasion to utter thanks for having grounded himself well in the mastery of these tools of the trade.

Elementary as it may seem, the reader of this issue must be sure that he knows his right hand from his left. I am reminded at this point of the story of the sergeant drilling a bunch of rookies who was disgusted because many of the new recruits did not possess this knowledge. He proceeded to have each man tie a bunch of hay to the left foot and a bunch of straw to the right foot, thus giving rise to the once-familiar marching order, "Hayfoot, strawfoot". In his section on the rule of debit and credit, Dr. Nelson emphasizes the fact that debit items are placed on the left-hand side of the account, while credit items are placed on the right-hand side. Remembering the lengths to which Dr. Graves went in Issue Three (pp. 169ff.) to help us see that the algebraic equation must be kept always in balance, we can appreciate somewhat more the careful balance which is always necessary in accounting. For every item appearing on the debit side of an account, there must be a corresponding entry on the credit side. Dr. Nelson helps us to understand this by showing the close correspondence of debit and credit to plus and minus.

All of the formulae proposed by the three authors in this issue are capable of being resolved by the same processes as have been stressed in Issues Three and Four, dealing with algebra. To understand the steps by which these equations are transformed, the reader will do well to review the articles by Dr. Graves, Dr. McGiffert, Dr. Dines, and the editor. In many instances, particularly in the lengthy transformations necessitated in the latter part of Dr. Nelson's article and in the article by Dr. Lockwood, we have merely indicated the steps which are to be taken, assuming on the part of the reader the ability to take the intermediate steps for himself.

The comparison used by Dr. Nelson

in showing the correspondence among the various methods by which depreciation may be figured (pp. 1037-1048) is particularly interesting in that he shows how these methods may be reduced to a common equation through the use of the differential calculus. A comparison of the several tables of depreciation with the diagram on page 1046 will well repay the reader. Incidentally, you may be interested to know that Dr. Nelson spent the better part of two days in computing the data on which the diagram is based. For each of the unassuming lines depicted, there were necessitated month-by-month computations for the period of 10 years involved—120 computations for each of the four lines, or 480 in all. To appreciate the fine points in Dr. Nelson's exposition, the reader should review the treatment of differentiation in Dr. Wiener's article (pp. 468-476).

The discussion of the break-even point by Dr. Lockwood again calls for an understanding of the method of figuring percentages. Since the figures involved are large, recourse to the slide rule or to logarithms in performing the necessary divisions is to be recommended. Indeed, there are few steps in the mathematics of business where a knowledge of how to use the slide rule will not be found of great assistance in obtaining quickly and easily the approximate answers desired.

While it must be admitted that the accountant, in performing the calculations necessary to a solution of a given problem, does not always say to himself "Now I must use some arithmetic; here is where I need some algebra; this is a place where I must employ the calculus", the reader will readily see that the employment of the various branches of mathematics becomes second nature to the expert and he automatically switches from

one field to another as the need arises. In actual practice, the accountant turns to a table in which the solutions have already been worked out and gratefully makes use of the findings without having to go through the intermediate steps. Even though he takes such a shortcut, he is doing so because he already understands the general principles which allow him to do so.

One might expect that accounting is a very exact form of mathematics. We often are amused when we hear of a bank staff kept long after hours unwrapping roll after roll of dimes to try to discover which package contained one too many or one too few coins. We know that the books of a bank must be made to balance exactly at the close of each day's business.

Guided by our authors, however, we discover that there are innumerable points in the field of accounting where approximations are brought into play. As Dr. Wright points out in his discussion of weighting (p. 1067), perhaps no two accountants would be in complete agreement as to the exact weights to be apportioned to various items, so that no two would come out with exactly the same advice after being called into consultation in a given case.

In the solution of some of the problems in this issue, the reader

may discover that his final answer varies by a cent or two from that reported by the authors. This may be due to the method used or the tables employed in arriving at the solution. As we have already suggested in connection with the work on logarithms and the problems in Issue Number Fifteen (insurance), employment of tables having one or two more or fewer figures than those supplied in the text may lead the solver to arrive at a slightly greater or less degree of accuracy. Even in the problem of computing interest, as Dr. Nelson points out in his discussion (p. 1016), one may have a choice of the method to be used. The answer will differ slightly depending on whether one takes a 365-day year or approximates with a 360-day year (based on 12 months of 30 days each) or resorts to any one of the formulae proposed for ready solution of the problem.

In all three of the articles in the present issue, names of individuals and of corporations have been used to give added clarity and reality to the situation. It should be understood that all of these are fictitious and that any resemblance to persons living or dead is entirely accidental. There is no thought, either on the part of the authors or of the publisher, to advertise or to call attention to the solvency or business operations of any actual concern.

ABOUT OUR AUTHORS

OSCAR S. NELSON was born in Georgetown, Montana, on July 30, 1896. He attended preparatory schools in Bozeman and Missoula, Montana, and then interrupted his education to serve in the United States Army in France during World War I. In October, 1919, he entered the Wharton School of Finance

and Commerce of the University of Pennsylvania, located at Philadelphia, receiving the degree of Bachelor of Science in Economics in 1923. Since that time, he has been a member of the faculty of the school, his present rank being Associate Professor of Accounting. He was awarded the degree of Master of Arts in 1926

and that of Doctor of Philosophy in 1930.

During the past twenty years, Dr. Nelson has had outstanding success as a teacher of accounting, and has served also as consultant in various enterprises where expert advice in accounting was required. In addition, he has contributed articles to technical journals of accounting and has served on committees of national organizations. He is a member of Beta Gamma Sigma (national honorary fraternity of collegiate schools of business), the Lenape Club of the University of Pennsylvania, the American Accounting Association, and the American Association of University Professors.

EDWARD NEEDLES WRIGHT received most of his education in the city of Philadelphia, where he was born in 1897. He entered the Wharton School in 1915, but interrupted his studies for a year during World War I to perform relief and reconstruction work in France under the supervision of the American Red Cross and the American Friends' Service Committee. He received the degree of Bachelor of Science in Economics in 1922, that of Master of Arts in 1923, and that of Doctor of Philosophy in 1930. He is at present Associate Professor of Accounting in the same institution.

At various times, Dr. Wright has taught special courses at Swarthmore College and for the Bell Telephone Company as well as for the Evening and Extension Schools of Accounts and Finance of his university. For several years, he served also as Associate Director of Admissions and also conducted investigation and writing of reports for the university's Industrial Research Department. Our

readers will be interested to know that Dr. Wright has acted as a special examiner for the United States Civil Service Commission in connection with examinations for the position of administrative analyst.

Dr. Wright is a member of Beta Gamma Sigma, the honorary scholastic fraternity in his field, and is a co-author of a textbook, *Interpretation of Accounts*.

JEREMIAH LOCKWOOD, another native of Philadelphia, also received the degrees of Bachelor of Science in Economics and of Master of Arts from the University of Pennsylvania. In addition, he is licensed as a Certified Public Accountant by the Commonwealth of Pennsylvania. He has served as a teacher in the Wharton School continuously since 1914, having been elevated to the rank of Professor of Accounting in 1939, and has taught in the Evening School and the Extension School since 1916.

Professor Lockwood's position in the accountancy field is testified to by the fact that his colleagues in the Pennsylvania Institute of Certified Public Accountants have honored him with the delegation of important committee tasks almost continuously since 1933. He is a member also of the National Association of Cost Accountants, the American Accounting Association, and the American Institute of Accountants.

Space will not permit the listing of all the books and magazine articles which Professor Lockwood has written during his long career as a teacher and expert accountant, but mention should be made of the fact that he is a co-author of books on *Modern Accounting Systems* and *Textile Costing—an Aid to Management*, and of a monograph on *Accounting*.

• THE MATHEMATICAL BASIS OF ACCOUNTING •

By Oscar S. Nelson, Ph.D.

ACCOUNTING may be defined as the art and science of (1) recording, (2) presenting, (3) interpreting, and (4) projecting the financial facts of an enterprise. Its objects are accomplished through the use of business papers, journals, ledgers, statements, analyses, and budgets. The science and method of mathematics is used throughout accounting. Since accounting deals with quantities and money values, arithmetical computations are everywhere necessary. Algebraic equations and formulae are used in maintaining the equilibrium of accounts, in valuing assets, in computing liabilities, and in determining income and expenses. Even analytic geometry and the calculus are employed in analyzing and presenting accounting data and in making cost computations and budget estimates.

**ACCOUNTING STATEMENTS
AND EQUATIONS**

Before proceeding very far with the subject of accounting, the reader will need to understand the various types of accounting statements and equations and the relationships which exist among them.

The balance-sheet equation

True to its origin in mathematics, modern accounting is based upon a mathematical formula, the balance-sheet equation, developed as follows:

At a given moment of time,

Let A = the total value of all assets the legal title to which is vested in the proprietor or owner of a given enterprise,

L = the total amount of all liabilities or legal obligations of the proprietor or enterprise,

C = the net amount of the capital investment of the proprietor (net worth, capital, net capital, and proprietorship are synonymous terms denoting the owner's equity in an enterprise at a given time);

Then $A - L = C$ (assets - liabilities = capital)

Ia

or $A = L + C$ (assets = liabilities + capital)

Ib

or $L = A - C$ (liabilities = assets - capital)

Ic

or $C = A - L$ (capital = assets - liabilities)

Id

No matter which form of the formula is used, the result is an algebraic equation of the first degree. The equation can be solved in all cases where there is only one unknown.

The truth of the equation is best demonstrated by the fourth form, Id, ($C = A - L$).

Illustrative Problem

A grocer, John Smith, owns and operates his own store. He began business at the beginning of the current year, when he paid \$10,000.00 for the store land and building, \$2,000.00 for the fixtures and equipment, and \$3,000.00 for his stock of groceries. As he did not have enough money to pay for everything outright, he signed a mortgage on the store in the amount of \$5,000.00 and asked the wholesaler to trust him to the extent of \$1,500.00 on his purchase of groceries. After all of the arrangements stated above had been made, John Smith had \$800.00 left in his bank account, which he decided to devote to the business. What was John Smith's net capital at the time he began business?

$$\text{Formula Id: } C = A - L$$

A (total assets):

Land and building	\$10,000.00
Store fixtures and equipment	2,000.00
Inventory of merchandise (Stock of groceries)	3,000.00
Cash in bank	800.00
	<u>\$15,800.00</u>

L (total liabilities):

Mortgage payable	\$5,000.00
Accounts payable	1,500.00
	<u>\$6,500.00</u>

Substituting in formula:

$$C = \$15,800.00 - \$6,500.00 = \$9,300.00.$$

Therefore, John Smith's net capital was \$9,300.00 when he began business.

The accountant presents accounting data such as these in the form of a balance sheet, which embodies the above equation, but in form Ib, ($A = L + C$). The balance sheet of John Smith as of the date he began business is presented in Fig. 1.

JOHN SMITH, Proprietor

Balance Sheet as of January 2, 1944

ASSETS		LIABILITIES AND CAPITAL	
Cash in bank	\$ 800.00	Accounts payable	\$ 1,500.00
Inventory of merchandise	3,000.00	Mortgage payable	5,000.00
Land and building	10,000.00	Total liabilities	\$ 6,500.00
Store fixtures and equipment	2,000.00	John Smith (Capital)	9,300.00
Total Assets	<u>\$15,800.00</u>	Total liabilities and capital	<u>\$15,800.00</u>

Fig. 1

The form of the balance sheet is the accountant's way of expressing the mathematical equation, $Ib (A=L+C)$, without the necessity of using the customary mathematical signs.

The balance sheet of a partnership and the balance sheet of a corporation both differ from the statement shown in Fig. 1 in the manner of setting forth the capital. Instead of having the item, "John Smith (Capital) \$9,300.00", a partnership, since it would have at least two proprietors, would have to have at least two capital accounts. On a partnership balance sheet, therefore, the capital might appear as in Fig. 2.

For a corporation, there would be a surplus account, containing the accumulated profit, and a capital stock account, containing the invested capital. Since a corporation is usually larger and requires more capital, bonds payable rather than mortgage payable

Net worth:

John Smith (Capital)	\$5,200.00	
Richard Brown (Capital)	<u>4,100.00</u>	\$9,300.00

Fig. 2

would appear as a liability. Furthermore, the capital structure usually is more complicated than is indicated by the balance sheet shown above.

Fig. 3 is an illustration of the balance sheet of a corporation.

TEST YOUR KNOWLEDGE OF THE BALANCE-SHEET EQUATION

- 1 Harry Johnson paid \$20,000.00 for a completely-equipped dairy farm. In part payment, he gave a \$9,000.00 mortgage on the farm. He has \$100.00 cash in the bank. What is the amount of his net capital?
- 2 George Marshall's total assets are \$50,000.00 and his net capital is \$35,000.00. What is the total amount of his liabilities?
- 3 Howard Wilson lost his record of his assets. His net capital is \$25,000.00 and his total liabilities are \$20,000.00. What is the total value of his assets?
- 4 From the following facts taken from the books of the Distilled Beverages Corporation, compute the net worth as of December 31, 1943. Also prepare a balance sheet:

Accounts and notes receivable	\$ 4,865,000.00	Land, buildings, and equipment	\$11,575,000.00
Accounts payable	583,000.00	Marketable securities	123,000.00
Accrued wages, interest, and taxes	764,000.00	Notes payable	2,050,000.00
Bonds payable	3,954,000.00	Prepaid insurance premiums	394,000.00
Capital stock, common	2,500,000.00	Other current liabilities	20,676,000.00
Cash	2,042,000.00	Surplus	3,328,000.00
Inventories	14,856,000.00		

- 5 William Brown, a dentist, rents office space in the center of the city. His dental equipment, which is practically new, cost him \$3,000.00. He has supplies on hand costing \$400.00, his patients owe him a total of \$600.00 for dental services rendered, and he has \$1,200.00 cash in the bank. He owes no bills. What is the amount of his net capital?

TEST YOUR KNOWLEDGE OF THE INCOME-STATEMENT EQUATION

- 6 Walter Johnson, an insurance agent, received a total of \$3,162.00 in commissions during the year. He also received \$512.00 as interest on securities owned and \$480.00 as rent on a house he owns. His expenses were: office rent, \$300.00; stenography, \$260.00; and traveling, \$625.00. What was his net profit, computed on a cash basis?
- 7 C. W. Hammer, the owner of a hardware store, wants to know what his net profit was for the year just closed.

He also desires an income statement to show his bank. The facts are as follows:

H. J. WALKER

		Income Statement for the year 1943	
Cash discount on purchases	\$ 6,570.00	Sales	\$100,000.00
Cost of merchandise sold	279,802.00	Cost of goods sold	75,000.00
Operating expenses	62,813.00	Gross profit on sales	\$ 25,000.00
Sales	348,000.00	Operating expenses:	
		Selling expenses	\$11,000.00
		Administration expenses	8,000.00
			19,000.00
		Net: Profit on sales	\$ 6,000.00
		Income from investments	500.00
			\$ 6,500.00
		Interest on mortgage	2,000.00
		Net profit	\$ 4,500.00

Fig. 5

- 8 C. W. Hammer now wants to be shown how the cost of merchandise sold was computed from the following information:

Inventory of merchandise, 1/1	\$ 76,000.00
Purchases of merchandise	275,000.00
Returned purchases	10,000.00
Inventory of merchandise, 12/31	66,198.00
Freight, parcel post, and express on purchases	5,000.00

- 9 The Sanitary Beverages Corporation submits the following facts for the year 1943:

Cost of goods sold	\$14,575,000.00	Other income	\$ 187,000.00
Interest expense	1,336,000.00	Operating expenses	40,804,000.00
Income taxes	263,000.00	Sales	58,102,000.00

Compute the amount of net profit and present the facts in the form of an income statement.

The proprietorship equation

In a balance sheet, the balancing figure is the amount of net worth or capital of the enterprise. The amounts of the various assets and liabilities are first set forth and the amount of capital computed according to the balance-sheet equation, Ib, ($A = L + C$). It is evident, then, that C varies and depends upon A and L . As A tends to increase or L tends to decrease, C tends to increase, and *vice versa*. Changes in assets and liabilities, therefore, are reflected as changes in capital except to the extent that they offset each other. Financial transactions cause changes in assets and liabilities and, hence, sometimes cause changes in capital. It is the changes in capital that we intend to analyze here. What kinds of transactions cause the amount of capital to increase and what kinds cause it to decrease? Four types of financial transactions and only four result in a change in capital:

- a *The investment of additional capital* in the enterprise by the proprietor. Such a transaction results in the increase of one or more assets; hence,

an increase in total assets without an offsetting increase in liabilities. Capital, therefore, is increased.

- b *The withdrawal of cash or other assets* from the enterprise for the personal use of the proprietor. Such a transaction results in the decrease of one or more assets; hence, a decrease in total assets without an offsetting decrease in liabilities. The amount of capital, consequently, is decreased.
- c *The sale of merchandise or other assets* for an amount greater than cost or the earning of some other type of income. Such a transaction results in the increase of assets or the decrease of liabilities by an amount greater than any offsetting changes in assets and liabilities. The net result must, therefore, be an increase in capital, because the equation, $Ib, A = L + C$, always holds.
- d *The incurring of an expense or loss.* This type of transaction results in the decrease of assets or the increase of liabilities (or a combination of both) by an amount in excess of any offsetting changes in assets and liabilities; hence, capital decreases by that amount.

As the types of transactions described under c and d usually are summarized in an income statement, their effects on capital are combined into a net figure. It may then be stated that the net profit or the net loss of an enterprise for a given period, in the absence of other changes, results in an increase or a decrease in capital by the amount of the net profit or loss. Except for the net profit or net loss shown by the income statement, therefore, the only changes in capital from one balance-sheet date to another are those resulting from the additional investments and withdrawals of the proprietor.

The proprietorship equation may now be stated:

$$\begin{aligned} &\text{capital (end of previous period) + additional investments - withdrawals} \\ &\quad + \text{net profit (or - net loss)} = \text{capital (end of current period)}. \end{aligned} \quad \text{III}$$

Illustrative Problem

Harvey Long is the owner and operator of a sawmill. His capital investment in his business on December 31, 1942, as shown by his balance sheet, was \$25,264.23. During the ensuing year, he made withdrawals of cash for living expenses amounting in total to \$3,215.80. On October 15, 1943, Mr. Long received title to a tract of timber land under the terms of his father's will. The tract had been purchased a year earlier at a cost of \$10,500.00. Mr. Long's net profit for the year, 1943, as revealed by his income statement, was \$2,967.11. Compute the amount of his capital on December 31, 1943.

$$\$25,264.23 + \$10,500.00 - \$3,215.80 + \$2,967.11 = \$35,515.54.$$

Therefore, the capital of Harvey Long on December 31, 1943, is \$35,515.54. The same amount would result from the independent computation of Mr. Long's capital on December 31, 1943, by using equation Id, ($C = A - L$).

The above facts and computation are best set forth in an accounting statement entitled "Statement of Proprietorship". It is prepared in the form shown in Fig. 6.

The statement of proprietorship shown in Fig. 6 is that of a single proprietor. If prepared for a partnership, the same information must be given for each partner and a totals column also should be provided.

If the information given in this illustration represented the total figures for a partnership in which three partners are interested, the statement of proprietorship might be as illustrated in Fig. 7.

A corporation, on the other hand, does not prepare a statement of proprietorship in the form of Fig. 7. As indicated earlier, the net capital of a corporation is made

HARVEY LONG

Statement of Proprietorship, 1943

Capital balance as of 12/31/42	\$25,264.23
Additional investments	10,500.00
	<u>\$35,764.23</u>
Withdrawals	3,215.80
	<u>\$32,548.43</u>
Net profit	2,967.11
Capital balance as of 12/31/43	<u>\$35,515.54</u>

Fig. 6

EVANS, PALMER & DOYLE

Statement of Proprietorship for the year 1943

	Totals	Evans	Palmer	Doyle
Capital balances, 12/31/42	25,264.23	12,121.15	8,417.18	4,725.90
Additional investments	10,500.00	2,500.00	3,750.00	4,250.00
	<u>35,764.23</u>	<u>14,621.15</u>	<u>12,167.18</u>	<u>8,975.90</u>
Withdrawals	3,215.80	1,000.00	1,100.00	1,115.80
	<u>32,548.43</u>	<u>13,621.15</u>	<u>11,067.18</u>	<u>7,860.10</u>
Net profit	2,967.11	989.04	989.04	989.03
Capital balances, 12/31/43	<u>\$35,515.54</u>	<u>14,610.19</u>	<u>12,056.22</u>	<u>8,849.13</u>

Fig. 7

up of capital stock and surplus. Since capital stock ordinarily does not change in amount each year, it is customary to prepare only a statement of surplus to accompany the balance sheet and income statement. If there is a change in capital stock during a given year, however, a separate statement should be prepared to show such change.

Assuming that a corporation has the net worth indicated in the two illustrations above, statements of capital stock and surplus, after the form* of Fig. 8 are required. The net worth of the corporation is the sum of its capital stock and surplus.

LONG and COMPANY, Inc.

Statement of Capital Stock, 1943

Capital stock, 12/31/42	\$22,500.00
Capital stock issued during year	10,500.00
Capital stock, 12/31/43	<u>\$33,000.00</u>

Statement of Surplus, 1943

Balance of surplus, 12/31/42	\$ 2,762.23
Net profit, year 1943	2,967.11
	<u>\$ 5,729.34</u>
Dividends declared	3,215.80
Balance of surplus, 12/31/43	<u>\$ 2,513.54</u>

Fig. 8

*Figures used in all cases are illustrative only

TEST YOUR KNOWLEDGE OF THE PROPRIETORSHIP EQUATION

- 10 On January 2, 1943, William Watson began business with an investment of \$10,000.00. During the ensuing year, he made no additional investments in the business, but withdrew \$3,000.00 for personal use. His net profit for the year was \$5,000.00. What was the amount of his net capital on December 31, 1943?
- 11 Lawrence Brown had a net worth of \$15,000.00 on December 31, 1942. He made no withdrawals during the year, but invested \$4,000.00 additional in the business. He had a net loss for the year of \$2,000.00. What was the amount of his net worth on December 31, 1943?
- 12 John Wilson and George Wilson are partners. On December 31, 1942, their balance sheet showed net worth as follows:

John Wilson (Capital)	\$20,000.00	
George Wilson (Capital)	<u>10,000.00</u>	\$30,000.00

During the year, 1943, they made a profit of \$5,000.00, which they shared equally. John Wilson withdrew \$3,000.00 from the business during the year, and George Wilson made an additional investment of \$2,000.00. The net worth of the business on December 31, 1943, was \$34,000.00. Prepare a statement of proprietorship for the year. What was the net capital of each partner on December 31, 1943?

- 13 The Weaver Machinery Corporation had a surplus of \$150,000.00 on December 31, 1942. Its income statement for the year, 1943, showed a net profit of \$15,000.00. Dividends of \$10,000.00 were declared and paid during 1943. (a) What was the amount of surplus on December 31, 1943? (b) What was the surplus on December 31, 1943, if the income statement showed a loss of \$5,000.00 instead of a profit of \$15,000.00? Prepare a statement of surplus in each case.

THE INTERRELATIONSHIP OF ACCOUNTING STATEMENTS

The balance sheet, the income statement, and the proprietorship statement are interrelated. The capital balance as shown by the balance sheet is the same amount as that shown by the proprietorship statement as of the same date. Also, the net profit in the income statement is the same amount as that included in the proprietorship statement for the same period of time. Since the three statements may be prepared independently of each other from the accounting data supplied by a set of books properly kept, the accountant may prove his work by noting whether the amounts indicated above are in fact in agreement. Furthermore, if some of the required facts are missing from the books, it may be possible to supply the deficiency by making use of the principle of interrelationship here set forth.

For example, a certain proprietor may have neglected to keep an adequate set of books and may lack the information necessary to prepare an income statement. He still can compute the amount of his net profit or loss for the current year, provided he can obtain certain key facts.

Illustrative Example

Gordon D. Tupper began business a year ago with a cash investment of \$6,000.00. During the year since then, he paid living expenses of \$2,500.00 but made no additional investments in the business. His bank account shows a present balance of \$400.00 and he owes unpaid

Gordon D. Tupper, Prop
Balance Sheet as of December 31, 1943

Assets				Liabilities and Capital			
Cash		400	00	Accounts payable		900	00
Inventory of merchandise		3600	00	Gordon D. Tupper (Cap.)		5575	00
Store fixtures and equipment \$2,750.00							
Less depreciation 275.00		2475	00				
		6475	00			6475	00

Fig. 9

bills totaling \$900.00. An inventory of the merchandise now on his shelves reveals that he has on hand merchandise which cost him \$3,600.00. He also owns the store fixtures and equipment, which he purchased for \$2,750.00 but which he estimates will be worthless at the end of 10 years. Can he determine the amount of his net profit or net loss for the year?

GORDON D. TUPPER

Statement of Proprietorship, 1943

- a First, he should prepare a balance sheet as of the close of the current year, as in Fig. 9.
- b Next, he should prepare a statement of proprietorship for the year, as in Fig. 10.
- c It is now evident that he made a profit of \$2,075.00 for the year, since it was necessary to add that amount in the proprietorship statement in order to arrive at the capital balance of \$5,575.00 which was revealed by the balance sheet.

Capital balance, 1/1/42	\$6,000.00
Withdrawals	2,500.00
	\$3,500.00
Net profit	2,075.00
Capital balance, 12/31/43	\$5,575.00

Fig. 10

The interrelationship of the balance sheet, the income statement, and the proprietorship statement is illustrated in Fig. 11.

The interrelationship of these three statements is the same in principle for the three types of business ownership: the single proprietorship, the partnership, and the corporation. For a

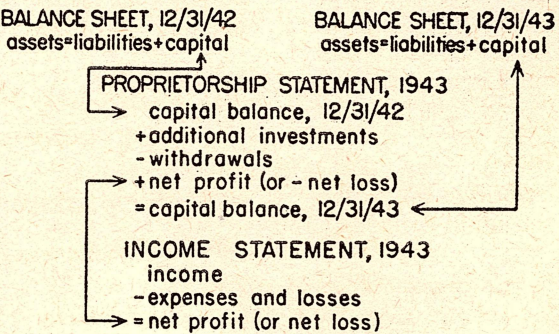


Fig. 11

partnership, the capital balances, the additional investments, and the withdrawals are set forth separately for each partner. For a corporation, the capital balances are made up of capital stock plus surplus; new capital stock issued constitutes the additional investments; and the amount of dividends declared and paid constitutes the withdrawals.

TEST YOUR KNOWLEDGE OF ACCOUNTING STATEMENTS

- 14 Wallace Jones does not keep a complete set of books. He wants to know if his net profit or net loss can be ascertained from the following facts: net capital, 12/31/42, \$22,000.00; net capital, 12/31/43, \$25,000.00; withdrawals, \$3,000.00; additional investments, \$1,000.00. Can his net profit or loss be computed? If so, what is its amount?
- 15 Howard Walker made no additional investments of capital in his business and withdrew no funds during the year, 1943. On December 31, 1942, his assets and liabilities were \$100,000.00 and \$40,000.00, respectively, while on December 31, 1943, they were \$110,000.00 and \$42,000.00. What was the amount of his profit for the year?
- 16 The business of Frank Davidson made a profit of \$13,000.00 for the year. His withdrawals were \$9,000.00. The business is now worth \$58,000.00. What was it worth a year ago?
- 17 The partnership of Scott & West has apparently been losing money during the year just closed. Information is not available to prepare an income statement. The partners know, however, that their capital a year ago was: Scott, \$10,000.00; West, \$15,000.00; total, \$25,000.00 and that withdrawals were: Scott, \$3,000.00; West, \$2,000.00; total, \$5,000.00. At the close of the current year, assets totaled \$48,000.00 and liabilities totaled \$30,000.00. Was there actually a loss? If so, how much? Assuming profits and losses are divided equally, prepare a statement of proprietorship for the firm.
- 18 The Johnson Corporation had a surplus of \$76,000.00 as of December 31, 1942. Its income statement for the year showed a net loss of \$4,000.00. However, a check in the amount of \$2,500.00 representing an over-assessment of federal income tax for a previous year was received from the Treasurer of the United States during the year. Dividends amounting to \$10,000.00 were declared. (a) What was the balance of surplus on December 31, 1943? (b) What was the net worth if capital stock amounted to \$175,000.00?

THE EQUILIBRIUM OF ACCOUNTS

Each of the three accounting statements—namely, the balance sheet, the income statement, and the proprietorship statement—embodies an equation which is always in a state of equilibrium. In the balance sheet, Ib,

$$\text{assets} = \text{liabilities} + \text{capital};$$

in the income statement, II,

$$\text{income} - \text{expenses} = \text{profit};$$

and in the proprietorship statement, III,

$$\begin{array}{ccccccc} \text{beginning} & + & \text{additional} & & & & \\ \text{capital} & & \text{investments} & - & \text{withdrawals} & + & \text{profit} \\ & & & & & & \text{(or - loss)} \\ & & & & & & \text{capital} \end{array} = \text{ending}$$

Furthermore, the three statements are interrelated as already indicated. Using the balance-sheet equation as a basis, we can combine the three equations into a single equation embodying all relationships:

$$\text{assets} = \text{liabilities} + \left[\begin{array}{l} \text{beginning} \\ \text{capital} \end{array} + \begin{array}{l} \text{additional} \\ \text{investments} \end{array} - \text{withdrawals} + (\text{income} - \text{expenses}) \right]. \quad \text{IVa}$$

In forming this equation, we replace the item, "ending capital", which normally belongs in the balance-sheet equation, by the items used in computing it in the proprietorship equation, except that the item, "net profit", is replaced by the items comprising it in the income-statement equation.

Since assets, liabilities, and capital constantly change from day to day, it is customary to set up accounting records, called *accounts*, in which to record the changes as they occur and from which to prepare the statements at periodic intervals. Normally such an account is opened for each asset, liability, proprietor's capital, proprietor's drawing, income, and expense item. Assuming that the beginning capital and the additional investments are to be recorded in the proprietor's capital account, the above equation may be restated as follows:

$$\begin{array}{l} \text{assets} \\ \text{accounts} \end{array} = \begin{array}{l} \text{liability} \\ \text{accounts} \end{array} + \begin{array}{l} \text{capital} \\ \text{accounts} \end{array} - \begin{array}{l} \text{drawing} \\ \text{accounts} \end{array} + \begin{array}{l} \text{income} \\ \text{accounts} \end{array} - \begin{array}{l} \text{expense} \\ \text{accounts} \end{array}. \quad \text{IVb}$$

By shifting the negative items to the left-hand side of the equation to make possible the use of the positive sign in all cases, we resolve the equation to the form,

$$\left. \begin{array}{l} \text{asset accounts} \\ + \text{drawing accounts} \\ + \text{expense accounts} \end{array} \right\} = \left\{ \begin{array}{l} \text{liability accounts} \\ + \text{capital accounts} \\ + \text{income accounts} \end{array} \right. \quad \text{IVc}$$

The equation is now in the form in which it appears in the ledger, or book of accounts, of an enterprise. The statement which includes all open accounts in the ledger at a given time is known as a *trial balance*. It embodies the above equation. The accounts which appear on the left-hand side of the equation are said to have debit balances and the accounts which appear on the right-hand side of the equation are said to have credit balances. The words, *debit* and *credit*, are technical accounting terms which, insofar as necessary here, mean *left* and *right* respectively.

The relationships of all basic accounting statements, equations, and accounts now may be set forth in a single chart (Fig. 12), which illustrates the relationship of statements, accounts, and equations of a single proprietorship or of a partnership. In the case of a corporation, the drawing accounts disappear and the capital accounts become capital stock and surplus accounts.

The accounts of an enterprise are used to record changes in the amount of assets, liabilities, capital, income items, and expenses.

These changes are brought about by entering into financial transactions. The accounting equations reflect conditions between transactions, since the values of all terms are taken as of a given instant of time. After each financial transaction, new values appear in the

TRIAL BALANCE			
BALANCE SHEET	Asset accounts	Liability accounts	assets - liabilities = capital
PROPRIETORSHIP STATEMENT	Drawing accounts	Capital accounts	beginning capital + additional investments - withdrawals + profit (or -loss) = ending capital
INCOME STATEMENT	Expense accounts	Income accounts	income - expenses = profit (or loss)
DEBIT BALANCES		CREDIT BALANCES	

Fig. 12

equations. The equations, however, must always maintain their equilibrium; hence, each financial transaction must change at least two accounts and the effects must counteract each other in order to maintain the equilibrium. This fundamental of accounting has led to the use of the term, *double entry*, in describing it. There is really not a double entry, but each transaction produces a double effect on the accounts. This double effect maintains the equilibrium of the accounts.

Illustrative Problem

John Brown owns and operates a small apartment house. As of January 2, 1944, his balance sheet appeared as shown in Fig. 13.

John Brown, Prop. *Balance Sheet as of January 2, 1944*

<u>Assets</u>				<u>Liabilities and Capital</u>			
Cash		3500	00	Mortgage payable	5000	00	00
Land and building	10000	00	00	John Brown (Cap.)	53500	00	00
	103500	00	00		103500	00	00

Fig. 13

Rents are collected on the first of each month for one month in advance and expenses are paid as promptly as possible. He prepares all statements on a cash basis. The accounts listed in Fig. 14 are kept in a small ledger. The transactions listed are those entered into during the month of January, 1944. The balance sheet as of January 31 is as shown in Fig. 15.

The reader will see from a consideration of these diagrams that the accounts are constantly changing, but that they are always in a state of equilibrium. Many of the accounts, however, are not changed each day as they would have to be to reveal up-to-the-minute facts, but are changed periodically because less bookkeeping is involved. For example, the rents were all collected on the first of the month and the rent earned account suddenly was increased by the entire amount.

CHART ILLUSTRATING EFFECT OF TRANSACTIONS ON ACCOUNTS

<i>Equation of Accounts Appears at Right</i>	ASSETS		+ EXPENSES = LIABILITIES +		INCOME + CAPITAL	
	<i>Cash</i>	<i>+ Land and Building</i>	<i>+ Expenses</i>	<i>= Mortgage Payable</i>	<i>+ Rent Earned</i>	<i>+ John Brown (Cap.)</i>
Balances, Jan. 1, 1944	3,500.00	100,000.00		50,000.00		53,500.00
a Rents are collected for month of January	A 1,400.00				A 1,400.00	
New account balances	4,900.00	100,000.00		50,000.00	1,400.00	53,500.00
b Interest on mortgage for one month is paid	D 250.00		A 250.00			
New account balances	4,650.00	100,000.00	250.00	50,000.00	1,400.00	53,500.00
c Installment on mortgage is paid	D 1,000.00			D 1,000.00		
New account balances	3,650.00	100,000.00	250.00	49,000.00	1,400.00	53,500.00
d Salaries and other expenses for January are paid	D 200.00		A 200.00			
New account balances	3,450.00	100,000.00	450.00	49,000.00	1,400.00	53,500.00
e Depreciation on building is recorded		D 300.00	A 300.00			
New account balances	3,450.00	99,700.00	750.00	49,000.00	1,400.00	53,500.00
f Income and expenses are transferred to capital			D 750.00		D 1,400.00	A 650.00
New account balances	3,450.00	99,700.00		49,000.00		54,150.00
g Proprietor withdraws cash for living expenses	D 500.00					D 500.00
Balances, Feb. 1, 1944	2,950.00	99,700.00		49,000.00		53,650.00

*A means to add to above amount and D means to deduct from above amount.

Fig. 14

The rents are really earned day by day, however. Expenses are treated in a similar manner although really incurred day by day. The capital account also is changed only at the end of the month for the entire amount of the net profit or net loss, although the profit or loss builds up day by day during the month. Before statements are prepared at the close of an accounting period, therefore, adjustments are usually necessary to set forth correctly the various assets, liabilities, expenses, and income items. The purpose and manner of these adjustments are discussed in a subsequent article (p. 1027 ff.).

TEST YOUR KNOWLEDGE OF TRANSACTIONS AND ACCOUNTS

- 19 Walter Gordon, a dentist, had a cash balance of \$1,750.00 and equipment worth \$3,500.00 as of January 2, 1944. During the year, he received in cash, fees for services rendered totaling \$6,400.00 and his expenses paid in cash were: salaries, \$1,300.00; rent, \$720.00; sundry, \$210.00. Allow \$300.00 depreciation on his equipment and compute his net profit.

John Brown, Prop.
Balance Sheet as of January 31, 1944

<u>Assets</u>				<u>Liabilities and Capital</u>			
Cash		2950	00	Mortgage payable		49000	00
Land and buildings		99400	00	John Brown (Cap.)		53650	00
		102650	00			102650	00

Fig. 15

- 20 Harold Wilcox owns and operates a garage. On January 1, 1944, his assets were: cash, \$1,200.00; inventory of cars, parts, etc., \$10,000.00; land, building, and equipment, \$25,400.00; and his liabilities were: notes payable, \$6,000.00; mortgage payable, \$10,000.00. During the year, he received \$8,640.00 for service and repairs to cars and paid \$5,810.00 for salaries and expenses. In addition, he used parts out of stock in repairing cars, costing a total of \$1,109.00. He also sold for \$4,000.00 cars which had cost him \$3,000.00. All cars sold were on hand at the beginning of the year, and he made no purchases of cars or parts during the year. All transactions were on a cash basis. (Assume that depreciation on building and equipment amounted to \$900.00 for the year.) What was the amount of Mr. Wilcox's net profit for the year?

THE RULE OF DEBIT AND CREDIT

The rule of debit and credit is a mathematical directive to be followed in recording financial transactions in journals and in ledger accounts. It is a rule which gives the direction and amount of the various changes in accounts resulting from a given financial transaction. Whereas "add" and "deduct" were used in the illustration in the previous section, in actual practice the words, *debit* and *credit*, become the direction signs. Debit means to place a given amount on the left-hand side of the account and credit means to place it on the right-hand side. Since placing an amount on the debit side of an

RULE OF DEBIT AND CREDIT

KIND OF ACCOUNT	DEBIT (Place on left side of account)	CREDIT (Place on right side of account)
Asset	To record an increase in the balance of the account	To record a decrease in the balance of the account
Liability	decrease	increase
Capital	decrease	increase
Drawing	increase	decrease
Income	decrease	increase
Expense	increase	decrease

Fig. 16

asset account tends to increase the account, while placing the amount on the left-hand side of a liability account tends to decrease the account, etc., it is necessary to state the debit-and-credit rule in a more or less elaborate fashion. Fig. 16 presents the rule stated in a comprehensive manner.

In addition to this rule, we should remember the fundamental that for every debit there must be an equal and corresponding credit or group of credits, and for every credit there must be an equal and corresponding debit or group of debits, or, more briefly, in the record of a given transaction, *the total of the debits must equal the total of the credits*. Note that all accounts that normally have debit balances are *debited for increases* and *credited for decreases* and that all accounts that normally have credit balances are *credited for increases* and *debited for decreases*.

The record of a transaction is first made in a journal and then transcribed or posted to the ledger accounts affected. Before making the journal entry, we must analyze the transaction to determine its debits and credits. Analyzing a given transaction involves:

- a Choosing suitable account titles,
- b Noting whether each account is an asset, a liability, an expense, an income, or a proprietorship item,
- c Determining the amount of increase or decrease in each account,
- d Applying the rule of debit and credit, and
- e Testing the equilibrium of the entry.

The form of the journal and ledger, together with illustrative entries (Figs. 17 and 18), are as follows:

Transaction

Adolph Sellers began business on January 2, 1944, investing the following:

Assets:

Cash	\$ 5,000.00
Inventory of merchandise	20,000.00
Auto trucks	6,000.00

Liabilities:

Accounts payable:	
D. O. Trust	\$ 7,000.00
Wholesale Stores Co.	12,000.00
Notes payable	4,000.00

Date	Accounts and Explanations	Folio	Debit	Credit
1944 Jan. 2	Cash		5000 00	
	Inventory of merchandise		20000 00	
	Auto trucks		6000 00	
	D. O. Trust			7000 00
	Wholesale Stores			12000 00
	Notes payable			4000 00
	Adolph Sellers (Cap)			8000 00
	Adolph Sellers began business on this date, investing assets and liabilities as above.			

Fig. 17

Cash									
1944	Jan.	2				5000.00			
Auto Trucks									
1944	Jan.	2				6000.00			
Wholesale Stores Co.									
							1944	Jan. 2	12000.00
Adolph Sellers (Cap.)									
							1944	Jan. 2	5000.00
Inventory of Merchandise									
1944	Jan.	2				20000.00			
Notes Payable									
							1944	Jan. 2	4000.00
D. O. Trust									
							1944	Jan. 2	7000.00

Fig. 18

TEST YOUR KNOWLEDGE OF THE RULE OF DEBIT AND CREDIT

- 21 (a) Prepare a ledger for J. A. Willis and record the following:
Balance Sheet, J. A. Willis, as of December 31, 1942

ASSETS		LIABILITIES	
Cash	\$ 7,500.00	Notes payable	\$ 50,000.00
Notes receivable	40,000.00	Accounts payable	17,000.00
Accounts receivable	58,300.00	Accruals payable	500.00
Inventory merchandise	60,000.00	Mortgages payable	50,000.00
Investments	23,200.00		
Prepayments	2,000.00		\$117,500.00
Real estate and equipment	180,000.00		
		PROPRIETORSHIP	
		J. A. Willis (Capital)	253,500.00
	<u>\$371,000.00</u>		<u>\$371,000.00</u>

Following is a summary of the transactions entered into by Willis during the year, 1943:

- a Lumber purchased on account, \$130,000.00; for cash, \$5,000.00.

- b Sales on account, \$160,000.00; for cash, \$10,000.00. The lumber sold cost \$133,000.00.
 - c J. A. Willis withdrew \$2,500.00 in cash; he later turned over to the business \$5,000.00 in bonds and \$10,000.00 in cash. He also took a salary of \$5,000.00.
 - d Received cash, \$118,300.00, and notes, \$25,000.00, from customers on account. Also received \$17,000.00 cash in payment of notes receivable.
 - e Creditors were paid \$105,500.00 cash on account and were given notes totaling \$25,000.00. Also paid off \$19,000.00 of notes.
 - f Expenses were paid in cash as follows: selling expenses, \$16,500; administrative expenses, \$4,000.00.
 - g Received \$1,000.00 in cash for interest on investments.
- (b) Also prepare a trial balance, an income statement, and a balance sheet assuming depreciation on buildings and equipment of \$2,700.00. Assume that J. A. Willis began business as of December 31, 1942, investing assets and liabilities as indicated.

22 The following additional transactions were entered into by Adolph Sellers during January, 1944: (Post to accounts in Fig. 18.)

- January 2—Sent check for \$2,000.00 to D. O. Trust on account; paid store rent for January, \$350.00, and office rent, \$50.00.
 4—Bought gasoline and oil for the trucks for cash, \$900.00.
 10—Cash sales for month to date, \$7,000.00.
 13—Paid Wholesale Stores Company \$4,000.00 on account.
 15—Paid salaries; delivery, \$180.00; sales, \$820.00; and administrative, \$250.00.
 18—Purchased merchandise on account from D. O. Trust, \$12,000.00.
 20—Cash sales from 11th to 20th, \$9,000.00.
 24—Paid \$2,000.00 note at bank; paid \$10,000.00 to D. O. Trust on account.
 31—Paid salaries as on the 15th; cash sales from 21st to 31st, \$8,000.00; paid store expenses, \$100.00, and office expense, \$55.00.

- (a) Record in a journal the entries for January and post to a ledger.
- (b) Prepare a list of ledger accounts and balances as of January 31, 1944.

It is clear from the foregoing discussion that accounting is based on mathematics. Its method is fundamentally a mathematical method:

- A In the basic-accounting equation, $\text{Id, capital} = \text{assets} - \text{liabilities}$, is found a dependent variable, capital, and two independent variables, assets and liabilities. It is clear that the amount of capital cannot change unless assets or liabilities or both change.
- B The changes in capital are explained by analyzing the asset and liability changes that cause them, and this analysis is set up in another equation.
- C The capital changes due to revenues and expenses are further analyzed and the result set up in the form of an income statement, which is a third equation.
- D The analyses are then combined into an equation of accounts and set up in such a manner that changes in every item can be recorded as transactions occur.
- E A mathematical rule is developed to facilitate and control the recording process so as to ensure accuracy and equilibrium throughout.

MATHEMATICAL COMPUTATIONS

In the preceding section, the basic applications of mathematics to accounting were considered—namely, those pertaining to the accumulating, recording, and presenting of accounting data. There are, in addition, numerous supplementary applications of mathematics to accounting. In the assembling of accounting data prior to entry upon the books, in the distribution of net profit after it has been determined, in the analysis of departmental operations, and in general office procedure, accounting computations of many different kinds are necessary. The present section deals with the computations and formulae of wide application. Most of the computations are of an elementary nature, such as trade discounts, cash discounts, bank interest and discount, anticipations, averaging accounts, partnership computations, etc. The basic interest formulae are then developed. These relate to compound interest, annuities certain, and sinking funds, an understanding of which is necessary on the part of accountants in income determination, balance-sheet valuation, and many other accounting computations. In the subsequent section, some of the more advanced mathematical computations are considered. Included are bond valuation and amortization, leasehold valuation and amortization, various depreciation methods, and mine valuation. The treatment of depreciation is of particular interest to mathematicians.

Trade discounts

Manufacturers, jobbers, and others frequently issue catalogues describing their wares and quoting prices. These catalogues ordinarily are for distribution to wholesalers and retailers for use in connection with the soliciting of sales to customers. Since the ultimate consumer is very likely to examine the catalogue when making a purchase, it is necessary that the price quoted in the catalogue be the maximum retail price that he is expected to pay. The manufacturer or jobber then supplies the wholesaler or retailer with a letter stating the per cent or list of percentages which the given wholesaler or retailer may deduct when ordering catalogue items.

The catalogue with its listed prices, together with the letter of discounts, called *trade discounts*, affords the manufacturer or other vendor a means of quoting prices to his immediate customers, whether they are wholesalers, retailers, or ultimate consumers. Furthermore, this device permits him to quote different prices to different customers without having to print different catalogues. Again, prices may be changed from time to time merely by changing the discounts allowed or by adding or taking away a discount. Trade discounts are not always a single discount percentage, but often comprise a series or chain of discounts, each of which is to be deducted in turn in arriving at the price to be paid, called the *invoice price*. Instances are known in the lumber trade, for example, where as many as forty discounts appeared in a chain of discounts from list price. Certain restrictions

on the granting of special discounts to large buyers have been imposed by the Robinson-Patman Act, however. These restrictions should, of course, be observed, wherever applicable. The Robinson-Patman Act was passed by Congress in 1936 as an amendment to the Clayton Act. It prohibits price discriminations unless based upon differences in the cost of distributing goods sold to competing customers. Where customers are in different categories, such as mail-order houses, jobbers, and retailers, distribution costs vary and price differentials are justified.

When only one percentage is given as a trade discount, the computation of the invoice price from the list price is relatively simple.

Illustrative Problem

If the list price or catalogue price of an article is \$100.00 and the trade discount is 40%, the invoice price or actual sales price is \$100.00 less \$40.00 or \$60.00. This latter amount is the only amount entered in the accounting records, the entry being as shown in Fig. 19.

In the event that a trade discount is quoted as several percentages—i.e., as a chain of discounts—each percentage is multiplied in turn by the amount remaining after the previous discount has been de-

ducted. To illustrate, assume that a lumber mill has quoted prices to a jobber as: list price, less 25-20-5-10. For a sale of lumber amounting to \$3,600.00 at list prices, the invoice price is then computed as follows:

List price	\$3,600.00
Less: 25% of \$3,600	900.00
	<u>\$2,700.00</u>
Less: 20% of \$2,700	540.00
	<u>\$2,160.00</u>
Less: 5% of \$2,160	108.00
	<u>\$2,052.00</u>
Less: 10% of \$2,052	205.20
Invoice price	<u><u>\$1,846.80</u></u>

Accounts receivable	6000
Sales	6000

Fig. 19

Thus, one discount percentage is applied at a time and the amount of the discount is deducted before the next percentage is applied, etc. It does not make any difference in what order the separate percentages are applied as long as the next one is applied against the reduced amount. In the case cited, the discounts might have been quoted as 5-20-10-25, but the result would have been the same. Test your knowledge by proving the truth of this statement.

It is sometimes desirable to combine a chain of discount percentages into one equivalent percentage. This can be done by assuming a list price of 100 and applying the percentages in turn in the orthodox manner and then deducting the resulting invoice price from 100. The result is a single percentage

that is equivalent to the chain of percentages. The chain of discount percentages just discussed would be combined as follows:

Assumed list price	100.00
Less: 25% of 100	25.00
	<u>75.00</u>
Less: 20% of 75	15.00
	<u>60.00</u>
Less: 5% of 60	3.00
	<u>57.00</u>
Less: 10% of 57	5.70
Resulting invoice price	<u>51.30</u>
100 - 51.30 = 48.70	

The 48.70, then, is the single percentage which is equivalent to the chain of percentages, 25-20-5-10. Now prove it by applying a trade discount of 48.7% against a list price of \$3,600.00 and determine whether the invoice price is the same—namely, \$1,846.80.

TEST YOUR KNOWLEDGE OF TRADE DISCOUNTS

- 23 A newly-married couple purchased a living-room suite from a retail furniture dealer at a price of \$175.00, the catalogue price. How much profit did the furniture dealer make on the sale if the wholesale dealer's price to him was list price, less trade discount of 10-10-5? What was the jobber's invoice price to the wholesaler if the trade discount was 20-10-10-5?
- 24 A retail radio store handles two different makes of radios. One wholesale dealer, A, quotes a trade discount of 30-20. The other wholesaler, B, quotes a trade discount of 5-20-25. Assuming that both makes of radio sell readily at the list prices, which is the more profitable line to the retail dealer?
- 25 A retail hardware dealer desires to dispose of some old stock of paint. He is willing to cut his price to a point where he will make a profit of 10%. What should he get for a gallon of paint with a list price of \$3.60 and a trade discount of 25%?
- 26 A retail toy dealer has been buying a certain toy from a wholesaler. The list price is \$25.00 and the trade discount 40-10. The wholesaler sends through another letter stating that an additional trade discount of 10 is added. As soon as his old stock is sold out, the retail dealer decides to pass on the last reduction in price to his customers. What should be the new price of the toy?

Cash discounts

Cash discounts, unlike trade discounts, appear upon the books of both the purchaser and the seller. Cash discounts must be recorded on the books since they do not become effective until and unless cash payment is made within a fixed period of time. While trade discounts are merely a device for quoting a price, cash discounts, ostensibly at least, represent a compensation for prompt payment of a bill.

When a sale or purchase transaction is entered into by an enterprise, the cash discount is set forth as a part of the terms of the transaction, as 2/10/30,

His checking account is then charged with \$1,000.00, the principal of the old note, and is credited with \$995.00, the proceeds of the new note.

The same procedure is followed month after month as long as the funds are desired.

A period longer than a month may, of course, be used, but the longer the discount period, the more costly it is for the borrower, since the discount is always deducted in advance. If funds are borrowed from a bank on collateral on, say, a demand basis, it is not necessary to discount a note each period. Instead, the full amount of the loan—e.g., \$1,000.00—is credited to the checking account of the borrower at the outset. The interest, \$5.00 per month or \$15.00 per quarter, as the case may be, is billed periodically and the loan runs until terminated by either the borrower or the bank.

When the period of time involved in interest computations is the year, the quarter, or the month, no problem is involved in finding the length of time. However, when the day is the unit of time, it must be decided whether interest is to be computed at $\frac{1}{365}$ of the annual rate or at $\frac{1}{360}$. Furthermore, it must be decided also whether the number of days actually elapsed is to be taken or whether one day is to be added to elapsed time, as is done by some banks. Unless there is an understanding to the contrary, the number of days elapsed should be used.

For example, the number of days from August 8 to August 27 on this basis is 19. This result is obtained by merely subtracting one date from the other. The *elapsed time* from August 28 to October 23 is 56 days, 3 days in August, 30 days in September, and 23 days in October.

One day added in each case gives *bank time* as used by some banks.

The question of using 365 days or 360 days as a year in obtaining the daily interest rate is one that will probably never be settled. It is claimed that using 360 days for a year is more convenient, and the result is approximately the same. If interest tables are used, however, it is just as convenient to use the exact rate, based on 365 days, as the ordinary rate, based on 360 days. Financial institutions always have available interest tables based upon 365 and 360 days, respectively.

TEST YOUR KNOWLEDGE OF INTEREST COMPUTATION BY THESE PROBLEMS

- 30 By the ordinary method, compute the amount of interest on a \$3,165.50 loan for 3 months at 5%.
- 31 Convert the above to exact interest.
- 32 Wallace Morton discounts his own \$1,200.00, 60-day note at the bank. Compute the amount of discount and prepare the necessary journal entry on his books.

- 33 William Watson discounts the note of a customer of his. The principal of the note is \$687.45, the discount rate 4%, and the number of days 31. Compute the amount of the discount.
- 34 A customer's 60-day note for \$1,870.40 dated January 14 is discounted at the bank on January 21. The discount rate is 5%. Compute the amount of the discount and prepare the necessary journal entry.
- 35 Compute the amount of ordinary interest for 6 months and 22 days at 4% on \$25,463.25. Prepare the journal entry recording the payment of the interest and the loan.
- 36 Verify the computation for problem 35 by reference to an interest table and also obtain the amount of exact interest.
- 37 Compute the ordinary interest on a \$1560.00 note bearing interest at 7%, dated April 14, 1943, and paid November 14, 1943, (a) by finding the exact number of days elapsed and (b) by finding the time in months. (c) Also compute the exact interest.
- 38 How many years are required for \$900.00 invested at $4\frac{1}{2}\%$ simple interest to earn \$101.25?
- 39 At what rate of simple interest must \$500.00 be invested to earn \$52.50 in 3 years?
- 40 If \$1000.00 is invested at $5\frac{1}{2}\%$ simple interest, how long will it require for the investment to double itself?

Discounting interest-bearing notes

It frequently happens that a business discounts some of its customers' notes to obtain cash for current operations. If these customers' notes (notes receivable) are non-interest-bearing, no mathematical problems are presented except those involved in discounting a *note payable* (*i.e.*, a note signed by the owners or officers of the business itself).

When a customer's interest-bearing note is discounted, however, additional computations are necessary. A customer's interest-bearing note increases in value daily because of the accruing interest. The maturity value of the note is then the principal amount because that is the amount the bank will receive when the note matures and is paid by the customer.

Illustrative Problem

The Ward Finance Company discounts two interest-bearing notes received from customers. The discount rate is 4%. One of the notes is dated June 20, the face amount is \$675.00, the interest rate 6%, and the time is 60 days. The other note is dated June 2, the face amount is \$500.00, the interest rate 6%, and the time is 60 days. Both notes are discounted on June 20. What amount of cash is obtained for each note?

As the first note is discounted the same day it is received, the discount

period is 60 days. The maturity value is computed as follows:

Face value	\$675.00
Interest at 6% for 60 days	6.75
Maturity value	<u>\$681.75</u>

The maturity value is then discounted at 4% for 60 days. The computation follows:

Maturity value	\$681.75
Discount at 4% for 60 days	4.55
Proceeds	<u>\$677.20</u>

As the second note has already run 18 days, the discount period is only 42 days. The maturity value is computed in the same manner as above—namely,

Face value	\$500.00
Interest at 6% for 60 days	5.00
Maturity value	<u>\$505.00</u>

The discount, however, is computed for only 42 days,

Maturity value	\$505.00
Discount at 4% for 42 days	2.36
Proceeds	<u>\$502.64</u>

The journal entries to record the discounting of the notes are illustrated in Figs. 26 and 27.

Cash	677.20	
Interest income		2.20
Notes rec. dis.	675.00	

Fig. 26

Cash	502.64	
Interest income		2.64
Notes rec. dis.	500.00	

Fig. 27

TEST YOUR KNOWLEDGE OF DISCOUNTING INTEREST-BEARING NOTES

- 41 William Reed discounted a customer's \$1,500.00, 90-day, 7% interest-bearing note at the bank. The note was dated June 30 and was discounted at 5% on July 10. Compute the amount of the discount and prepare the journal entry.
- 42 John Craig owes a bill of \$1,000.00 for merchandise purchased on terms of 2/10/30. He finds that he can discount at the bank a customer's \$1,000.00, 60-day, 7% note, having 20 days to run, just in time to pay the bill and take advantage of the cash discount offered. What would he gain or lose by entering into such an arrangement? (Bank discount rate, 6%.)
- 43 A \$5,000.00 note bearing interest at 6% is offered on August 1 in settlement of an account. The note is dated July 5 and matures on September 3. At what amount should it be taken?

Anticipations

Some large, well-financed merchandising establishments are in a position to deduct anticipations in addition to cash discounts when paying bills promptly. The amount of an anticipation on a given bill is computed by applying the current interest rate to the bill for the number of days by which the cash-discount date is anticipated.

Illustrative Problem

A given bank has a rule that a service charge of \$1.00 is to be made at the end of each month during which the average balance of a checking account is less than \$200.00. Determine whether the account reproduced in Fig. 29 is subject to the service charge for June.

A. P. Fellows

1943 June	2	Checks				75.00	1943 June	1	Balance				350.00
	9	Checks				100.00		5	Deposits				40.00
	17	Checks				120.00		29	Deposits				300.00
	25	Checks				80.00							
	30	Balance				315.00							
						690.00							690.00
							July	1	Balance				315.00

Fig. 29

DATE	AMOUNT	DAYS TO END OF MONTH	TOTAL DOLLAR DAYS
June 1—Balance	\$350.00	30	\$10,500.00
5—Deposit	40.00	25	1,000.00
29—Deposit	300.00	1	300.00
Less:			
June 2—Check	\$ 75.00	28	\$ 2,100.00
9—Check	100.00	21	2,100.00
17—Check	120.00	13	1,560.00
25—Check	80.00	5	400.00
		Net	6,160.00
			<u>\$ 5,640.00</u>

Average balance = $\$5,640.00 \div 30 = \188.00 .

Therefore, the average balance is less than \$200.00 and the service charge of \$1.00 is made for the month of June.

TEST YOUR ABILITY TO AVERAGE ACCOUNTS BY THESE PROBLEMS

- 46 The General Produce Company of Philadelphia acts as factor for numerous truck farmers. This company has been selling produce for John Jones but has been slow in remitting and, under his agreement, is required to pay interest at 6% on the average balance of the account for the month. The account appears in Fig. 31.

John Jones

1942 July	5	Remittance				1,000.00	1942 June	30	Balance				1,000.00
July	20	"				1,300.00	July	3	Produce				500.00
							July	16	"				800.00
							July	25	"				600.00

Fig. 30

Compute the amount of the remittance that will have to be made on July 31 to clear up the account.

- 47 Your bank makes a service charge of \$1.00 for each month that you do not maintain an average balance of \$200.00 in your checking account.

Your balance at the end of December was \$225.00. Checks drawn by you were presented at the bank as follows: January 5, \$50.00; January 15, \$60.00; January 25, \$10.00; and you deposited \$250.00 on January 29. Will the bank make a service charge for January?

- 48 One of the clauses of the partnership agreement of A and B provides that they are to be credited with interest at 6% on average capital balances as one step in the distribution of profits. The capital accounts for the year just ended appear in Fig. 31. Compute the amount of interest to be allowed each.

A (Capital)

1943	30	Withdrawal	500000	1942	31	Balance	10000000
Dec.	31	"	500000				

B (Capital)

1943	30	Withdrawal	300000	1942	31	Balance	60000000
Dec.	31	"	300000	1943	31	Additional investment	20000000

Fig. 31

BASIC MATHEMATICAL FORMULAE

Before entering upon a discussion of the problems of valuation and income determination, we find it necessary to develop certain basic mathematical formulae used in their solution. As these are basic to many computations, accountants must be thoroughly familiar with them even though tables are often available.

Compound interest and compound amount

Simple interest has been defined as the compensation allowed for the hire of money. It is computed for a single period of time, such as a month or a year, and the amount is paid to the creditor periodically. *Compound interest* results when the simple interest computed as above is not paid to the creditor at the end of the period, but, instead, is added to the amount of the principal and thus forms the basis for an increased amount of interest for the next period. The periodic addition of interest to the principal and the subsequent computation of interest on larger and larger amounts is known as *compounding*. Compound interest is the total amount of all interest amounts added to the principal over a series of periods. *Compound amount* is the accumulated total amount of principal and interest at the end of a series of periods.

Illustrative Problem

Compounding is easily explained by using an example. Lewis Ferguson deposited \$1,000.00 in a savings bank. The bank allowed interest at the rate of 3% and added the interest to his account at the end of each year.

The first year the interest was 3% of \$1,000.00, or \$30.00.

At the end of the second year, however, the bank added 3% of \$1,030.00, or \$30.90, to Mr. Ferguson's account.

The extra 90 cents was interest on the interest for the first year.

At the end of the third year, 3% of \$1,060.90, or \$31.83, was added; at the end of the fourth year, 3% of \$1,092.73, or \$32.78, was added; and at the end of the fifth year, 3% of \$1,125.51, or \$33.77, was added.

At the end of the fifth year, Mr. Ferguson withdrew his money to purchase war bonds. He received a total of \$1,159.28 from the bank.

The total, \$1,159.28, was the compound amount of \$1,000.00 at 3% compounded annually for 5 years.

The amount of compound interest received by Mr. Ferguson was \$1,159.28 minus \$1,000.00, the original principal, or \$159.28.

THE COMPOUND-INTEREST FORMULA

Compound interest and compound amount may be computed by means of a formula. This formula is developed as follows:

Let P = the principal, the original sum on which interest is computed,
 S = compound amount to which P will accumulate—*i.e.*, principal plus compound interest,
 n = the number of periods of compounding, or conversion (each period is usually a year or some fraction thereof),
 and i = the rate of interest for one period (one year if no other period is indicated).

At the end of the first period, the interest to be added will be Pi and the compound amount $P + Pi$. Therefore, at the end of the first period,

$$S = P + Pi = P(1 + i). \quad \text{VIa}$$

The amount of interest to be added at the end of the second period will be $P(1 + i)i$, and the compound amount at that time will be:

$$\begin{aligned} S &= P(1 + i) + P(1 + i)i \\ &= P(1 + i)(1 + i) \\ &= P(1 + i)^2. \end{aligned} \quad \text{VIb}$$

By the process of mathematical induction, we can show that the formula for the compound amount at the end of n periods is

$$S = P(1 + i)^n \quad \text{VIc}$$

Then, $P = \frac{S}{(1 + i)^n}$ = present worth of S , VII

and $P(1 + i)^n - P$ = compound interest. VIII

Applying the formula to the illustrative example, we obtain a result as follows:

$$\begin{aligned} S &= 1000(1.03)^5. \quad (\text{Formula VIc.}) \\ \log 1.03 &= 0.01284 \quad (\text{Table XI, page 127}) \\ &\quad \times 5 \\ &\quad \hline &\quad 0.06420 \\ \log 1000 &= 3.00000 \\ \log S &= 3.06420 \\ S &= \$1,159.30 \quad (\text{By interpolation}) \end{aligned}$$

The result is the same as before except that a slight inaccuracy resulted from the use of a three-place logarithm table.

Compound-interest tables may be used also in solving problems of this type.

The solution to the problem on page 1021 may be obtained by reference to a table. In the 3% column opposite to $n=5$, the compound amount of 1 is found to be 1.15927. Multiplying by P , which in this problem is \$1,000.00, we find the answer to be \$1,159.27. (Cf. Table LXXXII, p. 982, $(1.03)^n$ column.)

A formula to compute the present value of 1 at compound interest,

$$v^n = (1+i)^{-n}, \quad \text{IX}$$

may be developed from the compound-interest formula. The compound-interest formula is here used merely as a basis for the development of the annuity formula, the computation of bond values, etc.

THE AMOUNT OF AN ORDINARY ANNUITY CERTAIN

Another basic mathematical formula frequently used by accountants is the annuity formula. In its ordinary form, this formula may be stated as follows:

$$s_{\overline{n}|i} = \frac{(1+i)^n - 1}{i}. \quad \text{X}$$

$s_{\overline{n}|i}$ is a symbol used to denote the amount of an annuity certain of 1 per period accumulated at compound interest for a given number of periods. The letter, n , denotes the number of periods and the letter, i , the rate of interest per period. The amount accumulated at the end of n periods includes both the total of the n rents of 1 each and the compound interest on each rent to the end of the last period. The amount of an annuity of 1 in reality is the sum of a number of compound amounts obtained by placing 1 at compound interest at the end of each of a number of succeeding periods.

Assume, for example, that, at the end of the year, 1943, Harvey Long deposited \$1.00 in a savings bank at 3% compounded annually. A year later, he deposited another \$1.00 in the same account, and, at the end of the third year, still another \$1.00, etc., until he had deposited six \$1.00 amounts each one year apart. At the moment the last \$1.00 was deposited and before it earned any interest, the total in his account would be the amount of \$1.00 per annum at 3% compound interest for 6 years. It is important to note that the first \$1.00 was deposited at the end of the first year and that the amount of the annuity was computed at the moment the last \$1.00 was deposited. Although it is an annuity of six so-called rentals, the first was at compound interest for only 5 years, the second for 4 years, the third for 3 years, the fourth for 2 years, the fifth for 1 year, and the sixth for 0 years, illustrated below:

$$(1.03)^5 = 1.00000 + 0.03000 + 0.03090 + 0.03183 + 0.03278 + 0.03376 = 1.15927$$

$$(1.03)^4 = 1.00000 + 0.03000 + 0.03090 + 0.03183 + 0.03278 = 1.12551$$

$$(1.03)^3 = 1.00000 + 0.03000 + 0.03090 + 0.03183 = 1.09273$$

$$(1.03)^2 = 1.00000 + 0.03000 + 0.03090 = 1.06090$$

$$(1.03)^1 = 1.00000 + 0.03000 = 1.03000$$

$$(1.03)^0 = 1.00000 = 1.00000$$

$$s_{\overline{6}|3} = 1.00000 + 1.03000 + 1.06090 + 1.09273 + 1.12551 + 1.15927 = 6.46841$$

This illustration shows two important things:

First, it shows how each dollar is accumulated at compound interest from the time it is deposited until the date of valuation. For example, the compound amount of the first dollar is \$1.15927 five years later.

Second, the table shows that each year, in addition to the rental of \$1.00, interest is added, the amount of interest becoming larger each time. The amount of the annuity is \$6.46841.

The formula for computing the amount of an ordinary annuity certain is developed as follows:

Let $s_{\overline{n}|i}$ = the amount of 1 per period at compound interest,
 n = the number of periods,
 and i = the rate of interest per period.

Also bear in mind that each rental of 1 is paid in at the end of the period—*i.e.*, the first rental at the end of the first period, the second at the end of the second period, etc.—that i is the rate per period, and that a period may be a month, a year, or any other designated length of time.

Then $s_{\overline{n}|i} = (1+i)^{n-1}$, the first payment of 1, accumulated at compound interest for $(n-1)$ periods,
 $+ (1+i)^{n-2}$, the second payment of 1, accumulated at compound interest for $(n-2)$ periods,
 $+ \text{etc.}$, intermediate payments of 1 each,
 $+ (1+i)$, the next to the last payment of 1, accumulated at compound interest for one period,
 $+ 1.00$, the last payment of 1, paid in at the end of n periods and, hence, accumulating no interest.

The equation,

$$s_{\overline{n}|i} = (1+i)^{n-1} + (1+i)^{n-2} + \dots + (1+i) + 1, \quad \text{XI}$$

is a geometric progression in reverse, with a ratio between succeeding terms of $(1+i)$.

Now, multiplying each term on both sides of XI by $(1+i)$ produces

$$(1+i)s_{\overline{n}|i} = (1+i)^n + (1+i)^{n-1} + \dots + (1+i)^2 + (1+i). \quad \text{XII}$$

Then, subtracting equation XI from equation XII produces a third equation with all intermediate terms eliminated,

$$\begin{aligned} (1+i)s_{\overline{n}|i} - s_{\overline{n}|i} &= (1+i)^n - 1, & \text{XIIIa} \\ s_{\overline{n}|i}(1+i-1) &= (1+i)^n - 1, \\ s_{\overline{n}|i}i &= (1+i)^n - 1, \\ s_{\overline{n}|i} &= \frac{(1+i)^n - 1}{i}. & \text{XIIIb} \end{aligned}$$

Formula XIIIb, then, is the formula to be used in computing the amount of an annuity of 1 per period for n periods at compound interest at the rate i per period. It is easy to remember this formula by noting that the numerator, $(1+i)^n - 1$, is the formula for compound interest. Hence, the formula may be expressed in words as follows:

The amount of an annuity of 1 per period equals the compound interest of 1 divided by the rate.

The amount of an annuity of 1 per year for 6 years at 3% compound interest, as already computed in tabular form, may now be verified by formula:

$$s_{\overline{6}|} = \frac{(1.03)^6 - 1}{0.03} = \frac{1.194052 - 1}{0.03} = \frac{0.194052}{0.03} = 6.46841.$$

If the rental of an annuity is P , the amount of the annuity is found by first computing it for 1 and then multiplying by P .

$$\text{Rental} = P \cdot s_{\overline{n}|} \quad \text{XIV}$$

For example, in the case just considered, if the rental is \$2,500.00 per year, the amount is $\$2,500.00 \times 6.46841$, or \$16,171.00.

Annuity tables are available for use in the solution of annuity problems. The use of such tables often saves considerable time. A section of a table giving the amount of an annuity of 1 for various values of n and i is given on page 982.

Reference to this table in the $s_{\overline{n}|}$ column opposite 6 reveals that the amount of 1 per annum for 6 periods at 3% compound interest is 6.4684, the same as computed above.

Sinking-fund accumulation

One application of the mathematics of annuities is the preparation of sinking-fund schedules. Sinking funds are used frequently by municipalities to retire issues of bonds. Industrial, utility, and railroad companies also employ this means of retiring bonded indebtedness. The sinking fund may be established voluntarily or may be required by the bond indenture. In any case, periodic amounts are paid into a fund and the fund money invested in marketable securities or used to acquire outstanding bonds of the issue to be retired. Frequently a trustee is designated to receive and invest the funds. The annuity formula is used to make sinking-fund computations.

Illustrative Problem

Assume, for example, that a small city has just issued and sold \$100,000.00 of 10-year, $3\frac{1}{2}\%$ bonds to provide funds for a city hall. What annual amount will have to be placed in a sinking fund to retire all of the bonds at maturity, assuming that the sinking fund will earn an average of 3% through purchases of the city's own bonds or other marketable securities? Since \$100,000.00 is the amount to be accumulated,

$$P \cdot s_{\overline{10}|} = 100,000 = \frac{P(1.03)^{10} - 1}{0.03}$$

$$\text{and} \quad P = \frac{100,000 \times 0.03}{(1.03)^{10} - 1} = \frac{3000}{0.3439164} = \$8,723.05.$$

The result, \$8,723.05, is the sum of money required to be paid into the sinking fund at the end of each year for ten years in order to retire all bonds at maturity.

In solving for P in formula XIV, we may obtain the value of $(1.03)^{10}$ by ordinary arithmetic, by the use of logarithm tables, or by the use of a compound-interest table. If an annuity table is available, the work can be shortened somewhat by obtaining the value of $s_{\overline{10}|}$, or $\frac{(1.03)^{10} - 1}{0.03}$, directly and dividing 100,000 by it. The accountant

should master all of these methods, since he must be able to solve sinking-fund problems with or without the use of tables.

A sinking-fund schedule should be prepared so that the city officials will know at any time how much should be in the sinking fund and whether it is necessary to make up any deficiency. The sinking-fund schedule for the issue of bonds discussed above is presented in Table a.

TABLE a
SINKING-FUND SCHEDULE
10-year bonds, \$100,000 face value, 3% rate

YEAR	INTEREST ON SINKING-FUND BALANCE AT 3%	ANNUAL CONTRIBUTION TO SINKING FUND	TOTAL ADDITION TO SINKING FUND	AMOUNT IN SINKING FUND AT END OF YEAR
1		8,723.05	8,723.05	8,723.05
2	261.69	8,723.05	8,984.74	17,707.79
3	531.24	8,723.05	9,254.29	26,962.08
4	808.86	8,723.05	9,531.91	36,493.99
5	1,094.82	8,723.05	9,817.87	46,311.86
6	1,389.36	8,723.05	10,112.41	56,424.27
7	1,692.73	8,723.05	10,415.78	66,840.05
8	2,005.20	8,723.05	10,728.25	77,568.30
9	2,327.05	8,723.05	11,050.10	88,618.40
10	2,658.55	8,723.05	11,381.60	100,000.00

TEST YOUR KNOWLEDGE OF BASIC FORMULAE BY THESE PROBLEMS

- 49 Harold Keller is the beneficiary of a trust fund of \$10,000.00, which is to be invested and accumulated until he becomes of age. Assuming that the fund will earn 4% compound semi-annually, what will it amount to in 10 years?
- 50 How long will be required for a fund of \$5,000.00 invested at $4\frac{1}{2}\%$ compounded annually to amount to \$12,058.57?
- 51 At what rate of interest compounded semi-annually will \$2,000.00 amount to \$2,687.83 in 5 years?
- 52 Walter Morgan sets aside \$500.00 at the end of each year to build up a fund for his son's college expenses. Assuming that the fund is invested at 3% compounded annually, how much will be in the fund at the end of the fifth year?
- 53 A corporation desires to construct a new office building 10 years from now. The building will cost about \$150,000.00. Assuming that funds can be invested at 5% compounded annually, how much will have to be set aside at the end of each year to provide the necessary amount in 10 years?
- 54 A municipality issues \$200,000.00 of bonds due in 25 years. Assuming that sinking-fund money can be invested at 3% compounded annually, what is the amount of the annual sinking-fund contribution required to pay off the bonds at maturity?
- 55 Carl Lund desires to sell his right to receive a semi-annual distribution of royalties made by an estate owning oil lands. He has received

a relatively uniform amount averaging \$2,526.25 for the last 10 distributions. It is estimated that these distributions will continue for 20 years. If 8% interest is used as a basis for your calculation, what amount would you advise Mr. Lund to accept for his interest?

- 56 Mary Jones receives an annuity from an insurance company of \$200.00 per annum, beginning 1 year after the death of her husband. Since she is working, she deposits in a savings bank regularly the amount received from the insurance company. The savings bank allows interest at 2% compounded annually. (a) What amount does Mrs. Jones have in her account 2 years after her husband's death, immediately after depositing the \$200.00 received at that time? (b) What balance does she have 4 years after her husband's death?

INCOME DETERMINATION AND VALUATION

In order accurately to reflect operating results and financial condition, all of the items contained in an income statement, a balance sheet, and a statement of proprietorship (or surplus) must be carefully computed. If an item of revenue is overstated, net profit tends to be overstated by the same amount. If net profit is overstated, so is net capital, which is increased by the amount of the net profit. To offset the overstatement in net capital, some asset value is stated too high or some liability item is too low. Therefore, an error in income-determination, likewise, is an error in balance-sheet valuation. To obtain the correct amount for each revenue and expense item in the income statement and the correct value for each asset and liability item in the balance sheet, the accounts of an enterprise are adjusted periodically. The mathematics involved in making these periodic adjustments ranges from the simple arithmetic required to apportion fire-insurance premiums to the algebra and calculus required for certain valuations of fixed assets.

Periodic adjustments

As indicated above, periodic adjustments are necessary to determine correctly revenue and expense items and also to value correctly asset and liability items. Such adjustments are recorded on the books, with the result that the ledger accounts are brought up to date as of a given month or a given year, so that correct statements may be prepared on an accrual basis. These adjustments have the effect of changing account balances, just as ordinary entries have.

Certain revenue accounts are increased so as to include all revenue earned, whether received or not. Other revenue accounts are decreased to eliminate items of revenue already received but not yet earned. Certain expense accounts are increased so as to include all expenses incurred, whether paid or not. Other expense accounts are decreased to exclude expense items already paid but not yet incurred. In all cases of adjustment, balance-sheet accounts, also, are involved. If an expense account is increased, some asset account must be decreased

or some liability account increased. Likewise, a decrease in an expense account is offset by an increase in an asset account. In the same way, an increase in a revenue account requires an increase in an asset account or a decrease in a liability account, and a decrease in a revenue account is offset by an increase in a liability account.

Such adjustments should be made in every case where a ledger account does not accurately set forth the true facts as of the date on which the accounting period ends. Since conditions vary from one enterprise to another, it is difficult to classify the adjustments that are necessary. It is apparent, however, that every income-statement item, every balance-sheet item, and every net-worth item should be scrutinized to determine whether it is correctly set forth as of the date of preparation of the various accounting statements.

Many of the periodic adjustments require only simple mathematical computations. These will be considered first. The revenue from sales of merchandise is the difference between net sales and the cost of goods sold. The amount of ordinary sales usually is taken directly from the records without adjustment, except for returns and allowances. In determining the cost of goods sold, however, we must compute the value of the inventory of merchandise on hand and set it up on the books as an asset.

Taking an inventory merely means listing all items of merchandise on hand, together with quantities and prices, computing the values of the individual items, and adding to obtain the total value. In pricing each inventory item, we may use either the cost price, the present market price, or the lower of the two, provided the method chosen is consistently followed thereafter. Revenue from installment sales should be included in the income statement only in proportion to the cash actually collected on such sales. Likewise, installment accounts receivable should be included in the balance sheet only as the unrecovered cost of the merchandise sold. Revenues from sales to customers in foreign countries are often adjusted because of fluctuations in foreign-exchange rates, and the cash, accounts-receivable, and foreign-exchange accounts involved are revalued periodically in accordance with foreign-exchange rates.

The method most commonly employed in inventory pricing is *cost or market, whichever is lower*. In valuing an inventory by this method, we price each inventory item separately at the cost price or at the market price, whichever is the lower. The resulting total inventory valuation, therefore, is lower than that which would be obtained by using either cost prices or market prices exclusively.

Income from services—*e.g.*, rent, subscriptions, etc.—is measured on a time basis.

For example, magazine subscriptions received a year in advance are apportioned evenly over the months involved. An unearned subscriptions account is credited with the full amount received, and, at the outset, this full amount is a liability in the balance sheet. After the lapse of one

month, $\frac{1}{12}$ is set up as revenue and the liability is reduced by that amount.

Thus, the liability decreases month by month as the subscriptions are earned and included in the monthly income statements.

Revenue accruing from investments—e.g., interest on bonds owned—is measured in a similar manner.

For example, if a \$1,000.00, 4% bond is purchased at face value on October 1, the date of issue, $\frac{1}{12}$ of

\$40.00, or \$3.33, should be included in the income statement each month, even though the first interest coupon is not clipped until six months have elapsed. An asset account, likewise, should be set up and \$3.33 added each month as the interest is earned, as in Fig. 32. The asset account will have a debit balance of \$20.00 on the date the interest coupon becomes due, and at that time the entry will appear as in Fig. 33.

When bonds are purchased at a premium or discount, additional mathematical computations are involved. These computations are discussed in a subsequent section (page 1033). Certain expense accounts, also, are computed on a time basis, as will be readily understood.

For example, interest, taxes, and payroll are incurred as time elapses. At the end of an accounting period, these expense accounts must be adjusted by adding the amount incurred but not yet paid. If wages and salaries of \$500.00 are paid weekly and the accounting period ends in the middle of the week, the payroll for half of a week is incurred but not paid and the adjusting entry shown in Fig. 34 is necessary. The payroll account is thus increased to the total amount actually incurred and a liability set up for the unpaid amount. When the week's payroll is subsequently paid, the entry is made as in Fig. 35.

The result of the adjustment, therefore, is to apportion the payroll accurately between the two accounting periods involved and to provide a liability account for the balance sheet as of the date it is prepared. Adjustments are made for taxes and interest in a similar manner, except for interest on bonds payable when the bonds were sold at a premium or discount. These computations will be discussed later (page 1031).

Expenses which include the cost of materials and supplies are adjusted by taking an inventory of the materials and supplies on hand and setting up an asset for the amount of their value, the expense

[illegible]

Fig. 32

Cash	2000	
accrued interest on bonds		2000

Fig. 33

Payroll	25000	
Accrued wages and salaries		25000

Fig. 34

Accrued wages and salaries	250.00				
Payroll	250.00				
Cash				500.00	

Fig. 35

accounts being reduced by the same total. To accomplish this, we make a journal entry debiting materials and supplies inventory, and crediting the respective expense accounts. Expense items which are paid in advance—e.g., insurance premiums, rent, etc.—are apportioned to accounting periods on a time basis. When insurance premiums are paid, an asset account for prepaid insurance premiums is usually debited. At the end of each accounting period thereafter until the policies expire, an adjusting entry is made, setting up an expense account for insurance premiums and reducing the asset, prepaid insurance premiums. The journal entry in each case is to debit an insurance premiums account and to credit the prepaid insurance premiums account which was previously set up. The determination of expenses such as depreciation of buildings and equipment, depletion of oil land, and amortization of leaseholds requires the use of mathematical formulae of a more complicated nature. These are developed in a subsequent section (page 1037). In other respects, however, the procedure in making the adjustments is the same as outlined above. In each case, the journal entry is to debit an expense account and to credit a valuation reserve account.

TEST YOUR KNOWLEDGE OF ADJUSTMENTS

- 57 The Brown Clothing Store had an inventory at the beginning of the year of \$2,500.00. At the end of the year, the inventory was \$3,000.00. During the year, the purchases amounted to \$15,000.00, returns and allowances to \$200.00, freight and cartage to \$100.00, and cash discount to \$300.00. Compute the amount of the cost of goods sold.
- 58 The Scranton Dry Goods Company purchased during the year coal costing \$10,000.00. At the beginning of the year, coal was inventoried at \$2,000.00; at the end of the year, the inventory was \$1,500.00. What was the amount of the expense for coal used?
- 59 The Walton Manufacturing Company pays its employees every Tuesday for the week ending the previous Saturday. Accrued wages at the beginning of the year amounted to \$1,000.00; the amount paid during the year was \$55,000.00; and the amount accrued at the close of the year was \$800.00. What was the salary expense for the year?
- 60 The Long Life Insurance Company owns \$10,000 face value of 6% bonds on which the interest is payable April 1 and October 1. These bonds were purchased on April 1 of the current year. The books are closed on December 31. How much interest was earned in the current year?
- 61 The Jones Steel Company purchased a 3-year fire-insurance policy on July 1 for a premium of \$6,000.00. At the beginning of the year, the prepaid-insurance-premiums account had a balance of \$4,000.00, representing two-fifths of the original premium on a 5-year fire policy. The books are closed at the end of the year. Compute the current year's expense for insurance.
- 62 You are called in by a merchant who is very much concerned about his balance sheet and income statement for the year. An accountant had

installed books for him when he began business a year ago and his bookkeeper had prepared the statements from the books, which were kept in accordance with the instructions of the accountant. (Fig. 36.)

STATEMENTS PREPARED BY BOOKKEEPER—BEFORE ADJUSTMENT

Balance Sheet			
ASSETS		LIABILITIES AND CAPITAL	
cash	\$ 5,750.00	Accounts payable	\$14,800.00
Accounts receivable	10,000.00	Rent receivable in advance	1,200.00
Investments	5,000.00	Net worth	25,000.00
Store buildings	20,000.00		
Prepaid insurance	250.00		
	<u>\$41,000.00</u>		<u>\$41,000.00</u>

Income Statement			
Sales		\$50,000.00	
Cost of sales:			
Purchases		<u>42,000.00</u>	
Gross profit on sales		\$ 8,000.00	
Expenses:			
Selling expenses	\$12,000.00		
Administrative expenses	<u>6,000.00</u>	<u>18,000.00</u>	
Net loss on sales		\$10,000.00	
Other income		<u>1,200.00</u>	
Net loss		<u>\$ 8,800.00</u>	

Fig. 36

You explain the necessary adjustments to the proprietor and his bookkeeper as follows:

ADJUSTMENT ITEM	EFFECT ON BALANCE SHEET	EFFECT ON PROFIT AND LOSS STATEMENT	ADJUSTING ENTRIES	DEBIT	CREDIT
Inventory of merchandise	Increase an asset	Decrease cost of goods sold	Inventory of merchandise 12/31 To Cost of goods sold	13,000	13,000
Inventory of supplies	Increase an asset	Decrease an expense	Inventory of office supplies To Administrative expenses	100	100
Accrued income receivable	Increase an asset	Increase an income item	Accrued interest on investments To Interest on investments	75	75
Accrued expenses payable	Increase a liability	Increase an expense	Sales salaries To Sales salaries accrued	200	200
Deferred income (Credits)	Decrease a liability	Increase an income item	Rent received in advance To Rent income	400	400
Deferred expenses (Charges)	Decrease an asset	Increase an expense	Insurance expenses To Prepaid insurance	50	50
Reserve for depreciation	Decrease an asset	Increase an expense	Depreciation on buildings To Reserve for depreciation	300	300
Reserve for bad debts	Decrease an asset	Increase an expense	Bad debts To Reserve for bad debts	250	250

Prepare correct balance sheet and income statement.

Bond valuation

Bonds appear on balance sheets in two different places, as *bonds owned* on the asset side, and as *bonds payable* on the liability side. In both cases, it is necessary to make computations periodically to determine the value for balance-sheet purposes and the revenue or expense for income-statement purposes. Since the computations are the same in both cases, only the valuation of bonds owned is considered.

The balance-sheet value of a bond owned depends upon two things: the present value of the principal amount due at the date of maturity and the present value of the coupon amounts due periodically until the bond matures. In all cases, the value depends upon the rate of interest used, the time involved, and the principal and coupon amounts. If a bond is purchased at face value and is to be valued on the basis of the coupon rate of interest, its value, aside from accrued interest, will always be the face amount. Seldom, however, are bonds purchased at face value or valued on the basis of the coupon rate. A bond purchased at a discount and valued upon the basis of the yield rate will have a value below the face amount. The value, however, will steadily increase until it exactly equals the face amount at maturity. On the other hand, a bond purchased at a premium and valued on the yield basis will have a value above the face amount, but this value will decline periodically until it, likewise, will exactly equal the face amount at the date of maturity.

Illustrative Example A

A balance sheet is being prepared as of the close of a given accounting period. A \$1,000 bond is owned and is to be valued for balance-sheet purposes. The bond will mature in exactly 4 years. Interest is payable semi-annually at a coupon rate of 5% per annum, and the bond was purchased several years ago at a price to yield 6%. What is its value?

The present value of the principal amount of \$1,000.00 due in 4 years on a 6% semi-annual basis is found by using the compound interest present value formula or compound interest table (See p. 982, col. 3):

$$\frac{1}{(1.03)^8} = 0.78941,$$

$$0.78941 \times 1,000 = \$789.41.$$

The present value of the eight semi-annual coupons of \$25 each on a 6% semi-annual basis is found by using the formula or table for the present worth of an annuity (See p. 982, col. 5):

$$a_{\overline{8}|} = \frac{(1.03)^8 - 1}{0.03(1.03)^8} = 7.020,$$

$$7.020 \times 25 = \$175.50.$$

The present value of the bond is found by adding the two amounts above determined, namely:

Present value of principal	\$789.41
Present value of coupons	175.50
Present value of bond	<u>\$964.91</u>

The difference between the balance-sheet value and the maturity value of a bond at a given time is the amount of discount or premium. This bond is valued at a discount of \$35.09, the difference between \$1,000.00 and \$964.91. This discount is amortized over the remaining life of the bond. The amount

TABLE c
SCHEDULE OF BOND AMORTIZATION

\$1,000, 5% coupon bond purchased on 4% basis

PERIOD	BALANCE-SHEET VALUE AT BEGINNING OF 6-MONTHS' PERIOD	INTEREST AT 6% ON BEGINNING BALANCE-SHEET VALUE	COUPON, 5% ON FACE VALUE	AMORTIZATION OF PREMIUM	BALANCE-SHEET VALUE AT END OF 6-MONTHS' PERIOD	PERIOD
1st	\$1,019.04	\$20.38	\$25.00	\$4.62	\$1,014.42	1st
2d	1,014.42	20.29	25.00	4.71	1,009.71	2d
3d	1,009.71	20.19	25.00	4.81	1,004.90	3d
4th	1,004.90	20.10	25.00	4.90	1,000.00	4th

TEST YOUR KNOWLEDGE OF BOND VALUATION BY THESE PROBLEMS

- 63 A corporation issues \$100,000 of 5%, first-mortgage bonds, maturing in 10 years. Interest is payable semi-annually. The bonds are sold at a price to yield the investors $4\frac{1}{2}\%$. Compute the amount of bond premium.
- 64 Compute the amount of premium to be amortized at the end of the first period and the interest cost for the first period for the bond liability of problem 63.
- 65 An insurance company purchased a \$1,000,000 issue of bonds as an investment. The coupon rate was $5\frac{1}{2}\%$, payable semi-annually. The bonds are redeemable in 20 years at 110 and were purchased at a price to yield the insurance company 4% on its investment. (a) What was the balance-sheet value of the bonds at the time they were purchased? (b) What was the balance-sheet value a year later? (c) What was the income on the bonds for the year?
- 66 Using the bond table, determine the balance-sheet value of \$100,000 of bonds owned if the coupon rate is 5% payable semi-annually, the yield is 6%, and the bonds mature in 15 years.
- 67 On the asset side of a balance sheet is an asset, "Discount on bonds, \$4,265.10", representing the unamortized discount on \$100,000 of bonds payable. The coupon rate on the bonds is 5% payable semi-annually and the bonds have 5 years to run to maturity. (a) Compute the amount of discount that should be amortized at the end of the next 6-months' period. (b) Compute also the interest cost for the next 6 months on this issue.
- 68 A \$10,000 bond was purchased at a cost of \$10,138.46. It matures in 3 years and bears interest at the rate of 5% payable semi-annually. What is the yield to the investor?
- 69 A city offers for sale on January 1, 1944, an issue of bonds amounting to \$150,000 bearing interest at 3%, payable semi-annually. The bonds are to be retired serially, \$10,000 at the end of each year, beginning with December 31, 1944. What price should be paid to yield $2\frac{1}{2}\%$ on the investment?

TABLE d
TABLE OF BOND VALUES
Interest at 5%, payable semi-annually

NET COST OR YIELD	5 YEAR	10 YEAR	15 YEAR	20 YEAR
3%	109,222.19	117,168.64	124,015.84	129,915.85
4%	104,491.29	108,175.72	111,198.23	113,677.74
5%	100,000.00	100,000.00	100,000.00	100,000.00
6%	95,734.90	92,561.26	90,199.78	88,442.61
7%	91,683.40	85,787.60	81,607.96	78,644.93

Leaseholds

A leasehold represents a prepayment of a rental on real estate for a definite period. A leasehold may be acquired direct from the owner or purchased from a lessee with the approval of the owner. In either case, the value of the leasehold may be looked upon as the present worth of a periodic rental at an assumed rate of interest.

Illustrative Problem

If a manufacturer desires to obtain a 10-year lease on a plant and to pay the entire rental in advance, how much should he offer if the monthly rental is \$500.00 payable in advance and interest rates are around 6%?

The formula for the present worth of an annuity is

$$a_{\overline{n}|i} = \frac{1-v^n}{i} = \frac{(1+i)^n - 1}{i(1+i)^n} = \frac{s_{\overline{n}|i}}{(1+i)^n} \quad \text{XV}$$

However, since all rental payments are payable at the beginning of the period instead of at the end, the present value of a rental of 1 per period is the present value of an annuity due—namely

$$(1+i) \text{ times } a_{\overline{n}|i}, \text{ or } \frac{(1+i)^n - 1}{i(1+i)^{n-1}}, \text{ or } \frac{s_{\overline{n}|i}}{(1+i)^{n-1}} \quad \text{XVI}$$

Therefore, the present value of the leasehold is

$$\$500(1.005)a_{\overline{120}|0.005}, \text{ or } \$45,261.91.$$

Since the lease is to run for 10 years and the rent is payable monthly, 120 rental payments are involved. The interest rate is 6% per annum, or $\frac{1}{2}$ of 1% (0.5%) per month. From an annuity table, we find the value of $a_{\overline{120}|0.005}$ at 0.5% to be \$90.07345. This amount must be multiplied by 1.005, since all rentals are due in advance. The present value of an annuity due of 1 is, therefore,

$$\$90.07345 \times 1.005 = \$90.52382,$$

and

$$500 \times \$90.52382 = \$45,261.91, \text{ as above.}$$

(The same result is obtained by using the formula. The present value of an annuity due is $a_{\overline{n-1}|i} + 1$ —i.e., $a_{\overline{119}|0.005} = 89.52382$; adding 1 gives 90.52382 as above.)

AMORTIZATION

Another problem in connection with leaseholds is the problem of *amortization*—i.e., the computation of the amount to be charged to expense during a given month or year and also the value for balance-sheet purposes at periodic intervals. From the standpoint of the lessor, also, the amount to be included in the balance sheet as a liability and the amount of rent earned must be computed periodically. The leasehold is set up on the books of the purchaser as an asset at the amount paid. The entry is shown in Fig. 38. Likewise, the amount

If the conditions under which a leasehold is purchased or sold differ from those above, the mathematical result is different, but the same principles apply in all cases.

TEST YOUR KNOWLEDGE OF LEASEHOLDS BY THESE EXERCISES

- 70 The Quality Food Market acquired a leasehold by paying the present value on a 6% basis of 3 annual rentals of \$1,000.00 each, due in advance. What was the purchase price of the leasehold?
- 71 At what value should the leasehold in problem 70 appear as an asset on the balance sheet of the Quality Food Market at the end of one year (a) if computed on a straight-line basis; (b) if computed on an interest or annuity basis?
- 72 What amount should appear as rental expense in the income statement of the Quality Food Market for the first year after the purchase of the leasehold (a) if computed on a straight-line basis; (b) if computed on an annuity basis?
- 73 What amount should the Quality Food Market include in its income statement as interest income, if it desires to show \$1,000.00 as rental expense the first year?
- 74 If a 10-year leasehold is purchased for \$100,000.00, what annual rental is contemplated by the purchaser if interest rates are around 4%?
- 75 The White Manufacturing Co. has a lease which still has two years to run and on which no advance payments have been made. The annual rental is \$5,000.00, payable in advance. Since rentals have recently been rising, another manufacturing company offers to purchase the lease from the White Co., assuming liability for the two unpaid rentals and paying an additional amount based on a \$7,500.00 total annual rental and a 6% rate of interest. What is the amount of the offer?

Depreciation

Depreciation is an expense or a loss. Certain assets employed by an enterprise are in service for a considerable period of time. During that time, they decline in value because of wear and tear, action of the elements, obsolescence, inadequacy, and untoward events. To the extent that depreciation is anticipated and provided for in the accounts, it is treated as an expense; to the extent that it is unexpected, it is treated as a loss. In either case, it is a shrinkage in asset value and hence tends to decrease net capital.

Only certain assets are said to depreciate—namely, such assets as buildings, equipment, etc., which have been built or manufactured, which are used in the conduct of the enterprise and which are subject to physical deterioration or obsolescence. Land does not depreciate, but coal, oil, timber, and mineral land are subject to depletion—*i.e.*, the actual removal of the natural resource. Merchandise manufactured or purchased for sale is held for such a short time that ordinarily it is considered as not depreciating.

The problem of accounting is to anticipate the amount of depreciation and to record it on the books as an expense as it occurs. The

solution of this problem involves mathematics as well as engineering and economics. The various methods of apportioning the total depreciable value are advanced to meet special conditions or theories. The straight-line method is based on the assumption that depreciation is a function of time. The constant percentage of diminishing-value method is designed to equalize costs. Various interest methods have been developed in an attempt to introduce interest as a factor in the cost of using assets. The unit-cost method is based on the assumption that the cost of a unit of product should be the same as for a new machine. Production and revenue methods are designed to depreciate assets in accordance with the use of the assets. These methods are here considered to the extent that they involve mathematics. In all cases, the amount of depreciation apportioned is the same—namely, the cost of the asset, plus estimated demolition and removal charges, less estimated salvage revenue. The term, *scrap value*, as here used means the estimated salvage value less the estimated cost of removal and sale.

THE STRAIGHT-LINE METHOD

The simplest and most commonly used method of computing the amount of periodic depreciation is the straight-line method. The name, *straight line*, follows from the fact that, under this method, the amount of depreciation is the same for each accounting period. Consequently, when the data are plotted in relation to coördinate axes, a straight line results. The formula for computing the amount of depreciation under the straight-line method is developed as follows:

Let C = the cost of the depreciable asset at the time it is acquired,
 D = the periodic amount of depreciation,
 V = the book value of the asset at the end of any given accounting period,
 t = the age of the asset at the end of any given accounting period,
 n = the life of the asset—*i.e.*, the number of accounting periods over which the depreciable value of the asset is to be apportioned,
 d = the rate of depreciation—*i.e.*, the per cent of cost per accounting period,
 and S = the scrap value, trade-in value, or residual value of the asset at the end of its useful life or at the time it is scrapped, traded in, or otherwise disposed of.

$$\text{Then} \quad D = \frac{C - S}{n}, \quad \text{XVII}$$

$$d = \frac{D}{C}, \quad \text{XVIII}$$

$$\text{and} \quad V = C - Dt. \quad \text{XIX}$$

The amount of periodic depreciation depends upon the cost of the asset, the estimated scrap value, and the number of periods it is planned to use the asset.

Formula XVII is used to compute the amount of depreciation to be charged as an expense during a given accounting period. An entry is then made debiting a depreciation expense account and crediting the asset account, or a new account entitled "Reserve for depreciation". The rate of depreciation is obtained with formula XVIII. Formula XIX gives the book value of the asset as of any given date.

Illustrative Problem

The Penn Food Store uses auto trucks to deliver merchandise to customers. It follows the policy of trading in its trucks at the end of 3 years of use. A new truck is purchased on January 2, 1943, at a cost of \$2,000.00. It is estimated that the trade-in value at the end of 3 years is \$560.00. What is the amount of annual depreciation? What is the amount of depreciation per month?

Substituting in formula XVII,

$$D = \frac{C - S}{n}$$

$$= \frac{2000 - 560}{3} = 480$$

or

$$D = \frac{2000 - 560}{36} = 40.$$

Therefore, the amount of depreciation annually is \$480.00 and the monthly amount is \$40.00.

The rate of depreciation, d , is found by using the formula, $d = \frac{D}{C}$, and the result in this case is

$$d = \frac{480}{2000} = 24\% \text{ per annum (or } 2\% \text{ per month).}$$

To find the book value of the truck at the end of, say, 18 months, we use the third formula, $V = C - Dt$. Substituting,

$$V = 2000 - 40 \times 18 = \$1280.$$

In this formula, C and D are constants, t is the independent variable, and V is the dependent variable. The locus of a point whose equation is $V = C - Dt$ is a straight line. For the problem in question the only values of t and V of any significance are those

between 0 and 3 for t and 2000 and 560 for V . The segment of the straight line plotted for values within these limits is shown in Fig. 42.

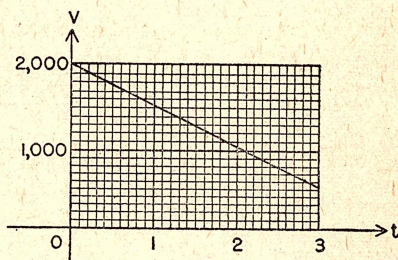


Fig. 42

Variations of the straight-line method are the units-of-output method and the productive-hours method. Under these methods, the total units of output and the total productive hours of an asset, respectively, are substituted for the life of the assets in periods of time.

TEST YOUR KNOWLEDGE OF STRAIGHT-LINE METHOD

- 76 Determine the annual amount of depreciation and the annual rate of depreciation for an auto truck, costing \$5,000.00, which it is anticipated will be traded in for another truck in 4 years at \$950.00.
- 77 Compute the book value of the truck at the end of 3 years.
- 78 A milling machine in a factory cost \$12,000.00. It is estimated that it will have a useful life of 10 years and a scrap value at the end of that time of \$500.00. Compute the amount of depreciation (a) for 1 year and (b) for 1 month. (c) Compute the book value at the end of $5\frac{1}{2}$ years.
- 79 A machine was purchased at a cost of \$50,000.00 on January 2, 1937. It was depreciated at the rate of 12% per annum. (a) What was the book value on December 31, 1943? (b) If it was sold at that time for \$5,000.00, was there a loss or a gain on the sale? How much? (c) What was the amount of depreciation per month?
- 80 A machine costing \$4,500.00 on January 2, 1939, was depreciated at the rate of 18% per year. On November 1, 1943, the machine was scrapped and sold. The cost of dismantling and disposing of the machine was \$250.00; the salvage amounted to \$600.00. Was there a gain or a loss on the sale of the machine? How much?
- 81 A manufacturing company purchased a machine for \$20,000.00, and, after depreciating it at the rate of 10% of cost per year for 8 years, rebuilt it at a cost of \$4,000.00. What was the book value of the machine at the end of the eighth year (a) before rebuilding; (b) after rebuilding?
- 82 Assuming that the machine after rebuilding has a remaining life of 5 years and a scrap value of \$400.00, compute the new rate of annual depreciation.

FIXED PER CENT OF BOOK VALUE METHOD

The most serious objection to the straight-line method of computing depreciation is that its use does not result in a uniform periodic cost for repairs and depreciation together, and the resulting book value of the asset does not approximate its trade-in value. Ordinary repairs are necessary to maintain a depreciable asset in good working condition. These repairs are light during the early life of the asset and become heavier as the asset grows older. To equalize the periodic cost of using an asset, depreciation should be heavier at first and become lighter each subsequent period. If this plan is followed, repairs and depreciation tend to complement each other, since the amount of the one increases period by period while the amount of the other decreases. Furthermore, it is a well-known fact that the trade-in value of an asset decreases more rapidly (absolutely) during its early life than later.

To meet these objections, a method of computing depreciation known as the fixed per cent of book value (or constant percentage of diminishing value) method has been devised. Under this method, the amount of depreciation of an asset is the greatest during the first

accounting period after acquisition and decreases for each subsequent period. The formulae involved are computed as follows:

Let C = the cost of the asset,
 D = the amount of depreciation for a given period,
 V = the book value of the asset at the end of a given period,
 t = the age of the asset at the end of a given period,
 n = the life of the asset,
 d = the rate of depreciation—i.e., the percentage applied to book value to obtain the amount of depreciation for a given period,
 and S = the scrap value.

The amount of depreciation for the first period is then the cost of the asset times the rate of depreciation, or

$$D_1 = dC.$$

For the second period, it is the book value ($C - D$) times the rate, or

$$D_2 = d(C - D_1) = d(C - dC) = dC(1 - d),$$

and, for the third period,

$$\begin{aligned} D_3 &= d(C - D_1 - D_2) = d[C - dC - dC(1 - d)] \\ &= dC[1 - d - d(1 - d)] = dC[1 - d - d + d^2] \\ &= dC[1 - 2d + d^2] = dC(1 - d)^2. \end{aligned}$$

Then, by mathematical induction,

$$D_t = dC(1 - d)^{t-1}.$$

XX

This is the formula to compute the amount of depreciation for any given period. Merely substitute the number of the period in place of t , the cost in place of C , and the rate in place of d .

The book value of the asset at the end of any given year also is determined by a formula. This formula is developed as follows:

At the end of the first period, the book value is the cost minus the depreciation for the first period, or

$$V_1 = C - dC.$$

At the end of the second period, the book value of the asset is its cost minus the depreciation for the first two periods, or

$$V_2 = C - [dC + dC(1 - d)] = C - dC - dC + d^2C = C(1 - 2d + d^2) = C(1 - d)^2.$$

Then, by mathematical induction, the formula for computing the book value at the end of any given period is

$$V_t = C(1 - d)^t.$$

XXI

At the end of n years, the book value and scrap value are the same; hence,

$$V_n = C(1 - d)^n = S.$$

The formula for computing the rate of depreciation, then, is developed from the preceding.

If $S = C(1 - d)^n$,

Then $(1 - d)^n = \frac{S}{C}$,

$$1 - d = \sqrt[n]{\frac{S}{C}},$$

$$d = 1 - \sqrt[n]{\frac{S}{C}}.$$

XXII

In solving depreciation problems under the constant percentage of diminishing value method, we usually find it necessary, or at least desirable, to use logarithms. This is especially true where we desire to find the rate of periodic depreciation.

Illustrative Problem

A manufacturing company purchased a special machine for \$25,000.00 and decided to depreciate it at the rate of 10% per annum on book value. What was the amount of depreciation the first year? The second? The tenth?

$$\text{Formula XX: } D_t = dC(1-d)^{t-1}.$$

The amount of depreciation the first year was 10% of \$25,000.00, or \$2,500.00.

The amount of depreciation the second year was

$$D_2 = 0.10 \times 25,000(1 - 0.10)^1 = 2,500 \times 0.90 = \$2,250.00.$$

The amount of depreciation the tenth year is computed by using logarithms.

$$D_{10} = 0.10 \times 25,000(1 - 0.10)^{10} = 2,500(0.9)^{10}$$

$$\begin{array}{r} \log 0.9 = 0.954243 - 1 \\ \text{Multiply by} \quad 10 \\ \hline 9.54243 - 10 \\ \text{Add log 2,500} \quad 3.39794 \\ \hline 12.94037 - 10 \\ \hline 2.94037 = \log \text{ of } 871.71 \\ \therefore D_{10} = \$871.71. \end{array}$$

TEST ON FIXED PER CENT OF DIMINISHING VALUE METHOD

- 83 A manufacturing company purchased a machine for \$5,000.00. The machine is estimated to depreciate to a trade-in value of \$840.35 in 5 years. The rate of depreciation is found to be 30% of book value each year. Compute the amount of depreciation for (a) the first year, (b) the third year, (c) the fifth year. (d) Now prove that the rate was computed correctly by showing that the book value at the end of 5 years is the trade-in value.
- 84 The Manchester Machine Manufacturing Company owns a lathe, costing \$3,000.00, estimated to have a life of 10 years. It is being depreciated at the rate of 20% of book value each year. What is the book value at the end of (a) 5 years, (b) 8 years? (c) What estimated scrap value was used in computing the 20% rate?
- 85 If a machine costing \$10,000.00 is depreciated each year at the rate of 11% of book value, in how many years will its value be reduced to the estimated scrap value of \$542.92?
- 86 Find the rate of depreciation by which a building costing \$2,400.00 must be reduced each year for 15 years so as to have a book value of \$400.00 at that time?
- 87 Determine the annual rate of depreciation to apply against the book value

- of an auto truck costing \$2,500.00 which it is planned to trade in at \$650.00 toward the purchase of a new truck at the end of four years.
- 88 A retail store has for 10 years been depreciating a warehouse at an annual rate of 10% on book value. What was the cost of the asset if book value is now \$6,972.00?
- 89 (a) Which is the more advantageous purchase, an auto truck costing \$2,000.00, lasting 4 years, and having a trade-in value of \$819.20, or one costing \$1,800.00, lasting 3 years, and having a trade-in value of \$1,029.22? (b) What is the saving in depreciation expense in the third year?

GEOMETRIC-PROGRESSION METHOD

The fixed per cent of book value method of computing depreciation, just discussed, results in a series of periodic depreciation charges which constitute a geometric series. The ratio between the amounts of depreciation for successive periods is constant. In the depreciation table (shown in Table e) for an asset costing \$1,000.00 and reduced to a book value of \$100.00 in 10 years, the ratio between any depreciation amount and the one immediately preceding it is always 79.433%.

Thus, $163.37 \div 205.63 = 0.79433$,
and $129.77 \div 163.37 = 0.79433$,
etc.

TABLE e
TABLE OF DEPRECIATION—FIXED PER CENT OF BOOK VALUE
Asset costing \$1,000 reduced to \$100 in 10 years; depreciation rate, 20.57%

YEAR	BOOK VALUE AT BEGINNING OF YEAR	DEPRECIATION ADJUSTMENT MADE AS OF END OF YEAR	BOOK VALUE AT END OF YEAR	ACCUMULATED DEPRECIATION TO END OF YEAR	YEAR
1st	1,000.00	205.67	794.33	205.67	1st
2d	794.33	163.37	630.96	369.04	2d
3d	630.96	129.77	501.19	498.81	3d
4th	501.19	103.08	398.11	601.89	4th
5th	398.11	81.88	316.23	683.77	5th
6th	316.23	65.04	251.19	748.81	6th
7th	251.19	51.66	199.53	800.47	7th
8th	199.53	41.04	158.49	841.51	8th
9th	158.49	32.60	125.89	874.11	9th
10th	125.89	25.89	100.00	900.00	10th

The series of periodic depreciation amounts shown in Table e, however, is but one of an infinite number of geometric series that can be set up to depreciate a \$1,000.00 asset to \$100.00 in 10 years. It is necessary merely to fix the ratio between successive depreciation amounts and then make the computations according to the formula for the sum of a geometric series. This formula is developed as follows:

Let C = the cost of the asset when acquired,
 S = the scrap value or trade-in value,
 D_1 = the amount of depreciation for the first year,
 r = the ratio of progression,
 and n = the number of periods over which the asset is depreciated.
 Then rD_1 = the amount of depreciation for the second period,
 and r^2D_1 = the amount of depreciation for the third period, etc.

TABLE f
TABLE OF DEPRECIATION—GEOMETRIC PROGRESSION

Asset costing \$1,000 depreciated to \$100 in 10 years; ratio of 0.75

YEAR	BOOK VALUE AT BEGINNING OF YEAR	DEPRECIATION ADJUSTMENT MADE AS OF END OF YEAR	BOOK VALUE AT END OF YEAR	ACCUMULATED DEPRECIATION TO END OF YEAR	YEAR
1st	1,000.00	238.43	761.57	238.43	1st
2d	761.57	178.82	582.75	417.25	2d
3d	582.75	134.11	448.64	551.36	3d
4th	448.64	100.59	348.05	651.95	4th
5th	348.05	75.44	272.61	727.39	5th
6th	272.61	56.53	216.03	783.97	6th
7th	216.03	42.43	173.60	826.40	7th
8th	173.60	31.83	141.77	858.23	8th
9th	141.77	23.87	117.90	882.10	9th
10th	117.90	17.90	100.00	900.00	10th

The total amount to be depreciated over the n periods—namely, $C-S$ —is then found by adding the terms in the geometric series as follows:

$$C-S = D_1 + rD_1 + r^2D_1 + \dots + r^{n-1}D_1,$$

$$C-S = \frac{D_1(1-r^n)}{1-r}. \quad \text{XXIII}$$

The amount of depreciation for the first period is found from formula XXIII by solving for D_1 , and is as follows:

$$D_1 = \frac{(C-S)(1-r)}{1-r^n}. \quad \text{XXIV}$$

The amount of depreciation for subsequent periods is found by multiplying XXIV by $r, r^2, r^3, \dots, r^{n-1}$.

The problem of depreciating an asset costing \$1,000.00 to a scrap value of \$100.00 in 10 years can now be solved in an infinite number of ways by merely choosing a ratio and then making the necessary computations. If it is desired that each succeeding depreciation amount bear the ratio of 0.75 to the preceding amount, the computation is as follows:

$$D_1 = \frac{(C-S)(1-r)}{1-r^n} = \frac{(1000-100)(1-0.75)}{1-(0.75)^{10}}$$

$$= \frac{900 \times 0.25}{1-0.056313} = \frac{225}{0.943687} = 238.4265.$$

Then $D_2 = 238.4265 \times 0.75 = 178.8199$, etc. as in Table f.

A geometric ratio of 0.9 between successive depreciation amounts results in the depreciation schedule shown in Table g.

The reader will see from these tables that varying the ratio of progression will permit the accountant to obtain any desired allocation of depreciation between periods.

By increasing the ratio, we may make the periodic amounts of

TABLE g
TABLE OF DEPRECIATION—GEOMETRIC PROGRESSION

Asset costing \$1,000 depreciated to \$100 in 10 years; ratio of 0.9

YEAR	BOOK VALUE AT BEGINNING OF YEAR	DEPRECIATION ADJUSTMENT MADE AS OF END OF YEAR	BOOK VALUE AT END OF YEAR	ACCUMULATED DEPRECIATION TO END OF YEAR	YEAR
1st	1,000.00	138.18	861.82	138.18	1st
2d	861.82	124.36	737.46	262.54	2d
3d	737.46	111.93	625.53	374.47	3d
4th	625.53	100.73	524.80	475.20	4th
5th	524.80	90.66	434.14	565.86	5th
6th	434.14	81.60	352.54	647.46	6th
7th	352.54	73.44	279.10	720.90	7th
8th	279.10	66.09	213.01	786.99	8th
9th	213.01	59.48	153.53	846.47	9th
10th	153.53	53.53	100.00	900.00	10th

depreciation as nearly equal as we desire. As the ratio approaches 100%, the results approach those obtained by using the straight-line method, which has already been discussed. By reducing the ratio, we may make the amount of depreciation charged off in the early periods as large as we desire. Thus, the geometric-progression method is more flexible than the fixed per cent of book value method, which is limited to one geometric series. Furthermore, the asset may be depreciated to zero under the geometric-progression method, while under the fixed per cent of book value method this is not possible. Table h is the depreciation schedule for depreciating a \$1,000.00 asset to 0 in 10 years using a progression rate of 90%.

TABLE h
TABLE OF DEPRECIATION—GEOMETRIC PROGRESSION
Asset costing \$1,000 depreciated completely in 10 years; ratio of 0.9

YEAR	BOOK VALUE AT BEGINNING OF YEAR	DEPRECIATION ADJUSTMENT MADE AS OF END OF YEAR	BOOK VALUE AT END OF YEAR	ACCUMULATED DEPRECIATION TO END OF YEAR	YEAR
1st	1,000.00	153.53	846.47	153.53	1st
2d	846.47	138.18	708.29	291.71	2d
3d	708.29	124.36	583.93	416.07	3d
4th	583.93	111.93	472.00	528.00	4th
5th	472.00	100.73	371.27	628.73	5th
6th	371.27	90.66	280.61	719.39	6th
7th	280.61	81.60	199.01	800.99	7th
8th	199.01	73.44	125.57	874.43	8th
9th	125.57	66.09	59.48	940.52	9th
10th	59.48	59.48	0	1,000.00	10th

SINKING-FUND METHOD

In accounting for depreciation, under ordinary circumstances, no thought is given to the future replacement of the depreciated asset. The aim is merely to amortize the cost of the asset over its useful life. It is, however, possible to create a sinking fund for the purpose of accumulating sufficient funds to replace an asset when it is discarded. The amount of cash that must be transferred periodically to the fund is computed as follows:

Let $C - S$ = the total depreciable value of the asset,

and n = the life of the asset.

Then $\frac{C - S}{s_{\overline{n}}}$ = the periodic contribution to the sinking fund. XXV

For example, a machine costing \$1,000.00 and having a scrap value of \$100.00 at the end of 10 years has a depreciable value of \$900.00. To accumulate \$900.00 in a sinking fund in 10 years, assuming an interest rate of, say, 5% compounded annually requires an annual contribution of \$71.56, which is computed as follows:

$$\frac{C - S}{s_{\overline{n}}} = \frac{1000 - 100}{s_{\overline{10}|5\%}} = \frac{900}{12.577} = \$71.56.$$

If the reserve for depreciation is to be increased each period by the amount of the addition to the sinking fund, the charge to depreciation for a given period will be the amount of the sinking-fund con-

tribution plus the amount of the interest on the sinking fund. The accumulation of the sinking fund and the building up of the reserve for depreciation are illustrated in Table i.

TABLE i
TABLE OF DEPRECIATION—SINKING FUND METHOD
Asset costing \$1000 depreciated to \$100 in 10 years; interest rate, 5%

YEAR	BOOK VALUE OF ASSET AT BEGINNING OF YEAR	ANNUAL DEPRECIATION CHARGE AND ADDITION TO RESERVE FOR DEPRECIATION			TOTAL IN SINKING FUND AND TOTAL IN RESERVE FOR DEPRECIATION AT END OF YEAR
		ANNUAL CONTRIBUTION TO SINKING FUND	INTEREST ON SINKING FUND	TOTAL DEPRECIATION CHARGE AND ADDITION TO SINKING FUND	
1st	1,000.00	71.55	0	71.55	71.55
2d	928.45	71.55	3.58	75.13	146.68
3d	853.32	71.55	7.33	78.89	225.57
4th	774.43	71.55	11.28	82.83	308.40
5th	691.60	71.55	15.42	86.98	395.38
6th	604.62	71.55	19.77	91.32	486.70
7th	513.30	71.55	24.34	95.89	582.59
8th	417.41	71.56	29.13	100.69	683.28
9th	316.72	71.56	34.16	105.72	789.00
10th	211.11	71.55	39.45	111.00	900.00
11th	100.00				

The sinking-fund method of computing depreciation is also an example of the geometric-progression method. The ratio of progression, however, is over 100% by the amount of the interest rate. In the case discussed above, the ratio of progression is 105%. XXIV can be used in obtaining the \$71.56 in the case above and the succeeding amounts may be obtained by multiplying by the ratio, 105%.

As a matter of fact, the geometric-progression formula is a universal formula applicable to nearly all depreciation computations. If the progression ratio is under 1, the result is a series of decreasing depreciation amounts. If the progression ratio is over 1, the result is a series of increasing depreciation amounts. As the progression ratio approaches 1, the result is a series of depreciation amounts approaching equality. By means of the calculus, we can show that, when the ratio of progression is 1, the depreciation amounts are equal and are the same as under the straight-line method.

In the formula,

$$D_1 = (C - S) \cdot \frac{1 - r}{1 - r^n}$$

the substitution of 1 for *r* results in the division of 0 by 0. However, if in the fraction, $\frac{1 - r}{1 - r^n}$, both the numerator and the denominator are differentiated before the substitution of 1 for *r*, the result is

$$D_1 = (C - S) \cdot \frac{1 - r}{1 - r^n} = (C - S) \cdot \frac{-1}{-nr^{n-1}} = (C - S) \cdot \frac{-1}{-n} = \frac{C - S}{n},$$

which is the formula for depreciation under the straight-line method.

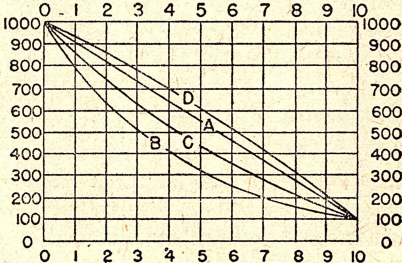


CHART ILLUSTRATING THE DEPRECIATION OF A FIXED ASSET FROM A COST OF \$1000 TO A SCRAP VALUE OF \$100 BY VARIOUS METHODS OF COMPUTATION
A Straight-line method
B Fixed per cent of book value method
C Geometric-progression method
D Sinking-fund method

Fig. 43

TEST YOUR KNOWLEDGE OF SINKING-FUND METHODS

- 90 Solve problems 86, 87, and 89 by using a geometric-progression ratio of 0.8 in each case.
- 91 Solve the illustrative problem used in the discussion of the sinking-fund method (page 1045) by using the geometric-progression formula and determine whether the same result is obtained.
- 92 Solve problems 86, 87, and 89 by the sinking-fund method using an interest rate of 4% compounded annually or a geometric-progression ratio of 1.04.

ANNUITY OR COMPOUND-INTEREST METHOD

The annuity or compound-interest method of determining the charge for depreciation is based on the principle that the expense of using the asset should include not only the amortization of the asset cost but also interest on the un-amortized book value of the asset. The total depreciation charge under this method, therefore, is the amount of depreciation under the sinking-fund method plus interest for the period on the remaining book value of the asset. There is no reason why interest on remaining book value cannot be treated as an expense in conjunction with other methods of figuring depreciation, but in practice it is generally applied as explained above.

It is not necessary to develop a new formula for the annuity or compound-interest method. The procedure involves merely the computation of the amount of depreciation for each period by the sinking-fund method and then the increase of each figure by the amount of interest for the period on the beginning book value of the asset. The rate of interest used in computing the amortization may be the same as or different from that used in computing the interest on book value. The method is illustrated in Table j. Note that certain of the figures are taken from Table i illustrating the sinking-fund method. The two methods may be contrasted by setting up the periodic journal entries for each method. The record at the end of the sixth year is shown by

Depreciation	9/32				
Sinking fund	9/32				
Reserve for depreciation				9/32	
Interest income				1977	
Cash				2/55	

Fig. 44

Depreciation	1/2/55				
Reserve for depreciation				9/32	
Interest income				3023	

Fig. 45

Depreciation	9/32				
Interest expense	3023				
Reserve for depreciation				9/32	
Interest income				3023	

Fig. 46

TABLE i
DEPRECIATION ACCUMULATION TABLE—COMPOUND INTEREST OR ANNUITY METHOD
Cost of asset, \$1000; scrap value, \$100; interest rate, 5%

YEAR	BOOK VALUE OF ASSET AT BEGINNING OF YEAR	DEPRECIATION CHARGE AT END OF YEAR	CREDIT TO INTEREST INCOME AT END OF YEAR. (RATE, 5%)	CREDIT TO RESERVE FOR DEPRECIATION AT END OF YEAR. (RATE, 5%)	TOTAL ACCUMULATION IN RESERVE FOR DEPRECIATION AT END OF YEAR
1st	1,000.00	121.55	50.00	71.55	71.55
2d	928.45	121.55	46.42	75.13	146.68
3d	853.32	121.56	42.67	78.89	225.57
4th	774.43	121.55	38.72	82.83	308.40
5th	691.60	121.56	34.58	86.98	395.38
6th	604.62	121.55	30.23	91.32	486.70
7th	513.30	121.55	25.66	95.89	582.59
8th	417.41	121.56	20.87	100.69	683.28
9th	316.72	121.56	15.84	105.72	789.00
10th	211.11	121.55	10.55	111.00	900.00
11th	100.00				

the entries in Fig. 44 and Fig. 45. It is not strictly correct to include the interest expense in the depreciation amount. A better plan is indicated in Fig. 46.

ARITHMETIC-PROGRESSION METHOD

Depreciation also may be computed by using an arithmetic progression. When this method is used, the amount of depreciation also can be varied period by period in an attempt to equalize costs in the face of a varying amount of repairs, etc.

Let D_1 = the amount of depreciation for the first period,
 d = the common difference between succeeding depreciation amounts,
and n = the number of periods.

Then $\frac{n}{2} [2D_1 - (n - 1)d] = C - S.$ XXVI

Now all that is necessary is to determine the life of the asset, n ; the depreciable value, $C - S$; and either D_1 or d or the relation between them. Table k illustrates the accumulation of depreciation by this method:

$$\frac{10}{2} [2D_1 - (10 - 1)9] = 1000 - 100.$$

It is well and proper to indicate at this point that the depreciation amounts for succeeding periods can be increasing rather than decreasing amounts if so desired.

TABLE k
DEPRECIATION ACCUMULATION TABLE
Cost of asset, \$1000; scrap value, \$100; $d = \$9$

YEAR	BOOK VALUE OF ASSET AT BEGINNING OF YEAR	DEPRECIATION CHARGE AT END OF YEAR	TOTAL ACCUMULATION IN RESERVE FOR DEPRECIATION AT END OF YEAR	BOOK VALUE OF ASSET AT END OF YEAR	YEAR
1st	1,000.00	130.50	130.50	869.50	1st
2d	869.50	121.50	252.00	748.00	2d
3d	748.00	112.50	364.50	635.50	3d
4th	635.50	103.50	468.00	532.00	4th
5th	532.00	94.50	562.50	437.50	5th
6th	437.50	85.50	648.00	352.00	6th
7th	352.00	76.50	724.50	275.50	7th
8th	275.50	67.50	792.00	208.00	8th
9th	208.00	58.50	850.50	149.50	9th
10th	149.50	49.50	900.00	100.00	10th

TEST YOUR KNOWLEDGE OF THE ARITHMETIC-PROGRESSION METHOD

- 93 A machine costing \$1,500.00 and having an estimated life of 5 years and a scrap value of \$300.00 is to be depreciated by the arithmetic-progression method. If the common difference between depreciation amounts is \$50.00, what is the amount of depreciation for (a) the first year? (b) The second?
- 94 If in example 93 the depreciation for the first year is \$250.00, what is the common difference between depreciation amounts?

UNIT-COST METHOD

Very often items of plant—e.g., machines and equipment—are continued in use long after improved models are available. It has been argued that it is not economical to discard an asset until it is fully depreciated. Recently, however, there has been a tendency to emphasize the income statement as an indicator of efficient management. This tendency has turned the spotlight on expenses and other items of cost and has led business men to consider whether cost can be reduced by discarding old and inefficient asset items even before they are fully depreciated.

The unit-cost method of valuing depreciable assets affords a means of determining whether a given asset item is valued at too high a figure in the light of the cost of using it in production as compared with the cost of using a new asset in a similar capacity. The unit-cost method is based upon the assumption that the cost of processing a unit of product through the use of a given asset should not exceed the cost resulting from the use of the most efficient asset available for the purpose. In comparing the costs of using an old and a new machine, for example, we must take several considerations into account, among which are the following:

- a The book value of each,
- b The periodic operating cost of each,
- c The output of each in units.

Let B = the book value of the old machine,
 C = the cost of a new machine,
 N = the number of periods the old machine will remain in service,
 n = the life of the new machine in periods,
 D = the periodic rent of an annuity to amortize B in N periods,
 d = the periodic rent of an annuity to amortize C in n periods,
 E^* = the periodic cost of operating the old machine, excluding depreciation and interest,
 e^* = the periodic cost of operating the new machine, excluding depreciation and interest,
 i = the interest rate,
 U = the number of units of output per period of the old machine,
 u = the number of units of output per period of the new machine.

and

* E and e may be broken down into elements if any of the costs are irregular from period to period or if any vary with production.

The cost of processing one unit on the old machine, then, is

$$\frac{D+E+Bi}{U} \quad \text{XXVII}$$

and the cost of processing one unit on the new machine is

$$\frac{d+e+Ci}{u}. \quad \text{XXVIII}$$

Now, if the cost of processing one unit on the old machine is not to exceed the unit cost of using an improved substitute, then the two fractions equal each other, and so

$$\frac{D+E+Bi}{U} = \frac{d+e+Ci}{u}. \quad \text{XXIX}$$

The periodic rent of an annuity to amortize B in N periods is found as follows:

$$D = \frac{B}{s_{\overline{N}|}}, \text{ where } s_{\overline{N}|} = \text{the amount of an annuity of 1 per period for } N \text{ periods at rate of interest } i.$$

In the same manner,

$$d = \frac{C}{s_{\overline{m}|}}.$$

Substituting for D and d in equation XXIX and solving for B results in the following:

$$\begin{aligned} \frac{\frac{B}{s_{\overline{N}|}} + E + Bi}{U} &= \frac{\frac{C}{s_{\overline{m}|}} + e + Ci}{u}, \\ B &= \frac{U}{\frac{1}{s_{\overline{N}|}} + i} \left(\frac{\frac{1}{s_{\overline{m}|}} \cdot C + e + Ci}{u} \right) - \frac{E}{\frac{1}{s_{\overline{N}|}} + i}, \end{aligned} \quad \text{XXX}$$

but

$$\frac{1}{s_{\overline{m}|}} + i = \frac{1}{a_{\overline{m}|}},$$

where $a_{\overline{m}|}$ = the present value of 1 per period at the rate of interest i ,

and

$$\frac{1}{s_{\overline{N}|}} + i = \frac{1}{a_{\overline{N}|}}.$$

Substituting in equation XXX produces the following:

$$B = U a_{\overline{N}|} \left(\frac{C \cdot \frac{1}{a_{\overline{m}|}} + e}{u} - \frac{E}{U} \right). \quad \text{XXXI}$$

Thus, the proper valuation of the old machine on the books, assuming that its value should be such that the cost of processing a unit of product using the old machine does not exceed what it would cost using a new, up-to-date machine, can be found by using formula XXXI.

Illustrative Problem

Suppose a manufacturing company owns a machine, having a remaining life of 4 years, which processes 1,000 units of product per year, and which costs \$1,500.00 a year to operate. A new, improved model of the machine can be purchased at a cost of \$5,000.00. The new machine has an estimated life of 10 years, will cost about \$2,000.00 a year to operate, and will process about 1,500 units of product annually. If current interest rates are around 4% per annum, what is the value of the old machine on this basis?

Using formula XXXI,

$$B = 1000a_{\overline{4}|i} \left(\frac{5000 \cdot \frac{1}{a_{\overline{10}|i}} + 2000}{1500} - \frac{1500}{1000} \right) = 3,629.90 \left(\frac{616.45 + 2000}{1500} - \frac{1500}{1000} \right) \\ = 3,629.90(1.7443 - 1.5) = \$886.79.$$

The value of the old machine is thus found to be \$886.79. If the actual book value is more than this amount, the difference should be written off against surplus, since depreciation for prior years has been inadequate.

TEST YOUR KNOWLEDGE OF THE UNIT-COST METHOD BY THESE PROBLEMS

- 95 Assume that the new machine in the illustrative problem just presented will process 1,600 units per year and compute the value of the old machine.
- 96 Assume an interest rate of 5% per annum instead of 4%, and then re-solve the illustrative problem and also problem 95.
- 97 If the cost of the new machine in problems 95 and 96 is \$10,000.00 instead of \$5,000.00, what is the value of the old machine in each case?
- 98 A factory having a book value of \$100,000.00 and a remaining composite life of 25 years has fixed overhead costs of \$10,000.00 per year, excluding depreciation and interest, and a capacity of 500 units of product. By spending \$15,000.00 on improvements, we can increase the capacity to 600 units of product per year. Assuming that no other changes would result and assuming an interest rate of 3% compounded semi-annually, determine the value at which the factory should be carried on the books.

Valuation of wasting assets

Wasting assets—such as mining property, timber land, and oil wells—are subject to depletion as the natural resource is removed. One method of valuing such property is to deduct the accumulated depletion since the beginning of operations from the original cost of the property. Depletion is computed by multiplying the cost of the property by a fraction whose numerator is the number of units removed from the property during the period and whose denominator is the total number of units the property will yield before it becomes exhausted.

When a wasting asset is to be purchased or sold, however, it must be valued upon a different basis. In determining its value, we find that the income that a property will produce is of primary importance.

Also to be considered are the rate of interest the investment should yield and the rental of the annuity necessary to restore the investment in the property by the time the natural resource becomes exhausted. A formula for valuing a wasting asset is developed below.

Let V = the value of the property,
 P = the periodic sinking-fund contribution necessary to accumulate the amount paid for the property, V , by the time the natural resources have been fully extracted,
 n = the number of periods,
 i = the investment rate per period,
 i^1 = the sinking-fund rate per period,
 and R = the periodic return from the operation of the mine, excluding depletion and interest on investment.
 Then $V = Ps_{\overline{n}|i^1}$, at interest rate i^1 ,
 $Vi = R - P$,
 $P = R - Vi$.

Therefore, $V = (R - Vi)s_{\overline{n}|i} = \frac{R}{\frac{1}{s_{\overline{n}|i}} + i}$, ($s_{\overline{n}|i}$ is at rate i^1). XXXII

Illustrative Example

A corporation owns and operates a coal mine. The mine produces 300,000 tons of coal annually and has an estimated life of 20 years at that rate. The return per ton of coal mined is about 30 cents, not considering depletion or interest on the investment. The corporation feels that an investment in this type of property should yield 7% annually, and that a sinking fund can be accumulated at a rate of about 4% compounded annually. The property will have no value when the mine is worked out. What is the value of the coal mine at present?

Using formula XXXII,

$$V = \frac{90,000}{\frac{1}{s_{\overline{20}|0.07}} + 0.07} = \frac{90,000}{0.033582 + 0.07} = \frac{90,000}{0.103582} = \$868,879.00.$$

The property, therefore, has a value at present of \$868,879 and an adjustment should be made for any variation from that amount.

TEST YOUR KNOWLEDGE OF WASTING ASSETS BY THESE PROBLEMS

- 99 Find the value of an oil well whose production is 500,000 barrels of oil per year and whose estimated life is 10 years. The current price of oil is \$3.00 per barrel and production costs are estimated to be \$1.50 per barrel. Assume a return of 10% per annum on the investment and a rate of 3% on sinking-fund money.
- 100 What is the value of a copper mine, the facts regarding which are as follows:

Estimated total production, 20,000,000 lb. of copper,
 Net return per pound of copper, 8 cents,
 Life of mine estimated to be 25 years,
 Rate required on investment, 12%,
 Sinking-fund rate, 4%?

• THE ANALYSIS AND INTERPRETATION OF ACCOUNTING STATEMENTS •

By Edward N. Wright, Ph.D.

ACCOUNTING is a means to an end, not an end in itself. The most detailed business records are of little value unless the information which they contain is summarized and presented in the form of statements. In turn, statements are chiefly of value when their contents are analyzed and the resulting facts are interpreted.

Accounting reports may be analyzed by various individuals and for numerous reasons. Executives, creditors, and investors all may have an interest in the statements of a business. Their analysis may be for the purpose of testing operating efficiency, for determining the desirability of extending credit, or for other reasons. Each type of analysis requires certain definite information and emphasizes particular facts and figures.

As analysis implies comparison, it is necessary for the analyst to know which facts are comparable and which are not. Furthermore, as most accounting information is expressed in dollars and cents, most analyses involve comparisons of figures. It is for this last reason that mathematics plays such an important part in the analytical phase of accounting.

PRE-REQUISITES FOR STATEMENT ANALYSIS

It is easy to minimize the difficulties and dangers involved in statement analysis and interpretation; even experts who have specialized in this phase of accounting for many years hesitate to draw conclusions from insufficient or confusing evidence. It is essential that an individual should have as broad a background as possible in the construction, valuation, and presentation of the accounts which he is called upon to analyze.

Ideally, the maximum amount of information about a business should be available before an analysis is undertaken, but analyses are so different in their nature and purpose that no standard requirement can be set. If the reports of a particular concern are under examination, the analyst should be familiar with the type of business, the nature of its product or service, its history and present location, and the character of those in charge. He should have its latest balance sheet, or, preferably, its financial and operating statements for several consecutive periods, certified as to their correctness by a public accountant. Any supplementary statements, schedules, or figures which aid in disclosing the true nature and status of the business should also be available. There is practically no limit to the amount

of general information which an analyst should have, particularly about business cycles and trade conditions and practices.

Unfortunately, not all of the information just mentioned is always available, even to the specialist. If sole reliance must be placed upon published statements, the analysis, of necessity, will be incomplete and unsatisfactory. Too often, printed reports are lacking in essential figures, or they are greatly condensed, or they contain confusing terminology.

Classification of accounts

Before analyzing a statement, we frequently find it helpful to simplify or *smooth* the figures and to rearrange and classify the accounts.

For example, it is much easier to deal with the figure, \$15,485,227.12, if the cents and hundreds of dollars are eliminated and the figure is written as

D. D. B. COMPANY

Balance Sheet as of December 31, 1943

ASSETS		LIABILITIES	
Cash on hand and in banks	\$ 460,102.24	Accounts payable—trade	\$ 501,196.27
Marketable securities	15,284.31	Notes payable	800,000.43
Notes receivable—trade	85,764.17	Accrued expenses	75,429.92
Accounts receivable—trade	1,324,444.13	First mortgage bonds	5,010,000.47
Inventories	2,125,747.35	Reserve for depreciation	
Land	8,242,600.00	Buildings	3,619,791.00
Buildings	10,941,067.00	Equipment	36,564.00
Equipment	412,701.00	Reserve for doubtful	
Patents, trade-marks, etc.	645,200.00	accounts	39,634.00
Deferred charges	80,822.20	Reserve for amortization	150,149.00
		Capital stock	11,120,495.00
		Earned surplus	2,480,472.31
		Capital surplus	500,000.00
Total assets	<u>\$24,333,732.40</u>	Total liabilities	<u>\$24,333,732.40</u>

Fig. 48

\$15,485,000. The amount, 227, in comparison with 15 million, is so small that for most analytical purposes it can be ignored. If the last figure to be smoothed or eliminated is a 4, 3, 2, 1, or 0, it is customary to allow the figure to the left to remain unchanged, but if the last figure is a 5, 6, 7, 8, or 9, the figure to the left is usually raised by 1. For example, the amount, \$2,380,472, would be simplified to \$2,380,000, but the amount, \$2,380,572, would become \$2,381,000.

Naturally, there is a limit beyond which it is not safe to go in the smoothing process; the extent of simplification depends on the degree of accuracy required in the final results.

The particular arrangement and classification of accounts depends largely on the type of analysis which is being made. For instance, if a bank is considering a loan to one of its depositors, it would want at least a separation of current and fixed assets and current and fixed liabilities on the balance sheet. In some types of analyses, net figures

are just as satisfactory as gross; for example, fixed assets may be shown on the balance sheet at their *book* or depreciated value rather than at cost. This is accomplished by deducting the reserves for

D. D. B. COMPANY

Balance Sheet as of December 31, 1943

ASSETS			LIABILITIES		
Current:			Current:		
Cash	\$ 460,000		Accounts payable	\$ 501,000	
Marketable securities	15,000		Notes payable	800,000	
Receivables, net	1,371,000		Accrued expenses	<u>76,000</u>	\$ 1,377,000
Inventories	<u>2,126,000</u>	\$ 3,972,000	Fixed:		
Land	\$8,243,000		First mortgage bonds		<u>5,010,000</u>
Buildings, net	7,321,000		Total liabilities		\$ 6,387,000
Equipment, net	<u>376,000</u>	15,940,000			
Deferred charges		81,000			
Intangibles, net		495,000			
				NET WORTH	
			Capital stock	*\$11,121,000	
			Earned surplus	2,480,000	
			Capital surplus	<u>500,000</u>	
			Total net worth		14,101,000
			Total liabilities and net worth		<u>\$20,488,000</u>
Total assets		<u>\$20,488,000</u>			

*This figure raised by \$1,000 in order to equalize the balance-sheet totals.

Fig. 49

depreciation on the asset side rather than showing such reserves on the liability side of the statement. Fig. 48 illustrates accounts and figures as they might appear before arrangement and simplification for analysis, while Fig. 49 shows the same figures rearranged.

BASES FOR INTERPRETATION

Before undertaking an analysis, the analyst should be aware of both the possibilities and the limitations inherent in the process. An *insider*, such as an executive, may be more familiar with a concern and have access to more detailed information than an *outsider*, such as a creditor or investor; on the other hand, the insider may have a certain bias or preconceived ideas which may affect his correct interpretation of the figures. In any case, it is important to remember that a mere analysis of the financial and operating *figures* of a business can give only a partial picture of a company's past history, present condition, and future prospects. Of necessity, the following sections of this article are limited to such financial analyses.

Percentages

A percentage or per cent is a proportional part of 100. Ten per cent means $\frac{10}{100}$ or 10 parts of 100. (See pp. 72-75.) In the analysis of

accounting reports, percentages are used in various ways to facilitate comparisons. In a 100% or *common size* statement, one figure is considered as the basic 100 and all other figures are expressed as per-

X. Y. Z. COMPANY

Consolidated Balance Sheet as of December 31, 1943

ASSETS			LIABILITIES		
Current	\$ 7,036,000	33.8%	Current	\$ 2,341,000	11.3%
Fixed	13,671,000	65.6%	Total liabilities	\$ 2,341,000	11.3%
Deferred	128,000	0.6%			
			NET WORTH		
			Capital stock	\$12,612,000	60.5%
			Surplus	5,882,000	28.2%
			Total liabilities and net worth	\$20,835,000	100.0%
Total assets	<u>\$20,835,000</u>	<u>100.0%</u>			

Fig. 50

centages of the base. Fig. 50 is an illustration of a condensed, simplified, percentage statement.

The percentage of 33.8 for the current assets is obtained by dividing \$7,036,000 by the base figure, \$20,835,000; the 65.6% for fixed assets is obtained by dividing \$13,671,000 by \$20,835,000, and so on for each percentage. (Use of a slide rule will save you time and trouble.)

Percentages are used also for comparing the figures of one period

X. Y. Z. COMPANY

Comparative Consolidated Statement of Earnings

	1942	%	1943	%
Net sales	\$20,398,000	100.0	\$18,517,000	100.0
Cost of goods sold	13,081,000	64.1	11,885,000	64.0
Gross profit on sales	\$ 7,317,000	35.9	\$ 6,632,000	36.0
Distribution, administrative, and general expenses	4,852,000	23.8	4,385,000	23.6
Net profit from operations	\$ 2,465,000	12.1	\$ 2,297,000	12.4
Other income	267,000	1.3	276,000	1.5
	\$ 2,732,000	13.4	\$ 2,573,000	13.9
Other expenses and losses	994,000	4.9	687,000	3.7
Net earnings available for dividends	\$ 1,738,000	8.5	\$ 1,886,000	10.2
Dividends on preferred stock	254,000	1.2	150,000	0.8
	\$ 1,484,000	7.3	\$ 1,736,000	9.4
Dividends on common stock	1,236,000	6.1	1,172,000	6.3
Balance to surplus	\$ 248,000	1.2	\$ 564,000	3.1

Fig. 51

with the figures of another period for the same company. Fig. 51 is an illustration of this type of comparison.

Percentages become even more useful when one company is compared with another, especially if the concerns are of different size, or if one company is compared with a group of companies. In Fig. 52, it is assumed that the X. Y. Z. Company wants to see how it stands in relation to other companies in the same industry.

**COMPARATIVE EARNINGS OF COMPANIES IN THE PULP AND PAPER INDUSTRY FOR
THE YEAR ENDING DECEMBER 31, 1942**

(In thousands of dollars)

	A.B.C. Co.	D.E.F. Co.	G.H.I. Co.	Totals
Net sales	\$3,195	\$16,466	\$270,759	\$290,420
Cost of goods sold	2,249	12,322	179,356	193,927
Gross profit on sales	\$ 946	\$ 4,144	\$ 91,403	\$ 96,493
Other operating expenses	690	3,481	59,832	63,903
Net profit from operations	\$ 356	\$ 663	\$ 31,571	\$ 32,590
Other income	33	25	2,584	2,642
	\$ 389	\$ 688	\$ 34,155	\$ 35,232
Other expenses	97	18	14,760	14,875
Net earnings available for dividends	\$ 292	\$ 670	\$ 19,395	\$ 20,357
Dividends paid	212	650	15,560	16,422
Balance to surplus	\$ 80	\$ 20	\$ 3,835	\$ 3,935

**EARNINGS OF THE X.Y.Z. COMPANY COMPARED WITH AGGREGATE EARNINGS OF OTHER
COMPANIES IN THE PULP AND PAPER INDUSTRY FOR THE YEAR ENDING
DECEMBER 31, 1942**

	X.Y.Z. Co.	Other Companies	Excess or Deficiency of X.Y.Z. Co.
			Excess Deficiency
Net sales	100.0%	100.0%	
Cost of goods sold	64.1	66.8	2.7%
Gross profit on sales	35.9	33.2	2.7%
Other operating expenses	23.8	22.0	1.8
Net profit from operations	12.1	11.2	0.9
Other income	1.3	0.9	0.4
	13.4	12.1	1.3
Other expenses	4.9	5.1	0.2
Net earnings available for dividends	8.6	7.0	1.5
Dividends paid	7.3	5.7	1.6
Balance to surplus	1.2%	1.3%	0.1%

Fig. 52

TEST YOUR KNOWLEDGE OF PERCENTAGES BY THESE EXERCISES

- 1 If a company sold \$1,240,000 of merchandise during a certain period but had \$3,000 of the sales returned by customers, would you consider \$1,240,000 or \$1,237,000 as the proper 100% base for calculating other percentages on the income statement? Why?
- 2 A company's December 31 figures were: current assets, \$17,512,583.30; fixed assets, \$586,133.46; deferred charges, \$284,449.54; other assets, \$389,240.32; current liabilities, \$5,018,878.47; fixed liabilities, \$12,600,000.00; and net worth, \$1,153,528.15. Construct a percentage statement.
- 3 After simplifying the figures in the preceding question to even thousands of dollars, construct another percentage statement. Note how much or how little difference the smoothing process makes in the percentages.
- 4 From the following accounts, prepare an income statement in proper form and calculate percentages for each item: net sales, \$28,851,000; cost of goods sold, \$26,682,000; other operating expenses, \$306,000; other income, \$5,000; other expenses, \$28,000; and dividends declared and paid, \$120,000.
- 5 If an income statement showed net sales of \$1,000,000 and a final net

loss of \$10,000, could the loss be expressed in percentage form? If so, how?

- 6 You are told that a company with net sales of \$472,620 paid total dividends of 1.5% and carried forward to surplus a balance of 3.4%. What were the actual amounts of the dividends and net profit carried to surplus?

Ratios

A ratio is the relationship between two figures or the proportion of one thing to another. It is calculated by dividing one figure by the other. The comparison may be expressed by placing the sign, :, or the word, *to*, between the figures, or it may be expressed as a fraction by showing one figure as the numerator and the other as the denominator. (See pp. 83-87.)

Illustrative Example

On December 31, a business had current assets of \$309,000 and current liabilities of \$103,000.

The relationship between these two facts may be expressed as

current assets, \$309,000 : current liabilities, \$103,000

or as current assets, \$309,000, to current liabilities, \$103,000

or as current assets, \$309,000 ÷ current liabilities, \$103,000,

or as
$$\frac{\text{current assets, \$309,000}}{\text{current liabilities, \$103,000}}$$

If desired, a ratio may be reversed by exchanging the numerator and the denominator, thus:

$$\frac{\text{current assets, \$309,000}}{\text{current liabilities, \$103,000}} \text{ or } \frac{\text{current liabilities, \$103,000}}{\text{current assets, \$309,000}}$$

In practice, the numerator is mentioned before the denominator; therefore, the "ratio of sales to receivables" means

$\text{sales} \div \text{receivables}$

whereas the "ratio of receivables to sales" means

$\text{receivables} \div \text{sales}$.

In a ratio, the result obtained by dividing one figure by the other may be expressed as a whole number (or fraction), as a proportion, as a decimal, or as a percentage.

Thus, the ratio,

current assets, \$309,000, to current liabilities, \$103,000,

may be expressed as 3, 3 to 1, 3.00, or 300%. The ratio,

current liabilities, \$103,000, to current assets, \$309,000,

may be expressed as $\frac{1}{3}$, 1 to 3, 0.333, or $33\frac{1}{3}\%$.

TEST YOUR KNOWLEDGE OF RATIOS BY THESE EXERCISES

- 7 Calculate and express in three different ways the ratios of net profit, \$4,200, to sales, \$114,000; current assets, \$16,017,000, to current liabilities, \$7,085,000; and fixed assets, \$611,470, to total assets, \$2,331,607.
- 8 Which would be the better from the standpoint of a creditor, a 300% ratio of current assets to current liabilities or a 30% ratio of current liabilities to current assets?
- 9 What would be the converse of the following ratios: (a) 200%, (b) 500%, (c) $66\frac{2}{3}\%$, (d) $12\frac{1}{2}\%$, (e) 150%?
- 10 Suppose that a company with current assets of \$200,000 and current liabilities of \$100,000 is seeking a short-term loan of \$50,000 from its bank. If we assume that the borrowed money will be kept as a current type of asset, would the loan, if made, change the ratio of current assets to current liabilities? If so, what effect would it have?

Averages

In mathematics, an average is a single figure which is representative of a group of figures.

THE ARITHMETIC MEAN

Probably the average used most commonly in accounting is the arithmetic average, or arithmetic *mean*. This average is calculated by adding the figures in a group or series and dividing the sum by the number of figures involved. (See pp. 75-77.)

*Illustrative
Example*

2.08
2.33
3.37
3.39
3.52
4.07
4.21
4.25
4.26
4.32
4.35
4.45
4.48
4.53
4.53
4.60
4.67
4.87
5.30
5.40
5.50
5.53
5.54
5.64
5.70
5.80
5.89

THE MODAL AVERAGE

Another average which is sometimes employed is the modal average, or *mode*. The mode is the figure in a group or series which occurs most frequently. This average has little significance unless the number of figures involved is large; also, it is necessary sometimes to calculate an approximate or theoretical mode if no figure in a group appears more than once. In determining the mode, we find it convenient to arrange the figures in an ascending or a descending order.

In the illustration, the figure, 4.53, is the mode because it is the only figure which appears more than once.

THE MEDIAN

A third type of average which is also used in analytical accounting is the *median*, or middle figure in a group or series. After arranging the figures

27)122.58
4.54

in an ascending or descending order, we find the median by adding one to the entire number in the group and dividing the sum thus obtained by 2.

In the illustration, there are 27 figures.

$$27+1=28(\div 2=14).$$

The median, therefore, is the fourteenth figure in the group, counting from either end, or 4.53. In this case, the median and the mode happen to be the same figure. If there is an *even* number of figures in a group, an imaginary or theoretical median can be calculated by finding the sum of the two middle figures and dividing by 2. Thus, in the illustration, if the final figure of 5.89 were omitted from the group, there would be 26 figures, of which 4.48 and 4.53 would be in the middle.

$$4.48+4.53=9.01(\div 2=4.505),$$

the theoretical median.

TEST YOUR KNOWLEDGE OF AVERAGES BY THESE EXERCISES

- 11 Following are the current assets to current liabilities ratios of 46, 52, 64, 69, and 83 concerns in the furniture industry over a five-year period: 394%, 366%, 346%, 302%, and 304%. Calculate the average number of companies reporting and the arithmetic mean of the five ratios.
- 12 Thirty-one companies in the same industry reported their average collection period (in days) as follows: 101, 86, 76, 65, 63, 63, 66, 59, 58, 55, 64, 65, 58, 56, 53, 55, 60, 50, 48, 55, 55, 49, 45, 42, 48, 52, 49, 48, 51, 50, 55. After arranging the figures in an ascending or a descending order, calculate (a) the mean, (b) the median, and (c) the mode.
- 13 Eliminate from the list of figures in the preceding question the first one mentioned, 101, and recalculate the imaginary or theoretical median and the arithmetic mean. Notice which average is affected more by the elimination of the one figure.

METHODS OF ANALYSIS

The mathematical devices used most commonly in analysis are percentages, ratios, and averages, and the various analytical methods in which they are employed are referred to as the trend-percentage method, the ratio method, the business-index method, and the sources and application of funds method.

Trend-percentage method of analysis

In the trend-percentage method of analysis, the original amounts in a time series are reduced to percentage form. This procedure emphasizes significant trends and relationships and makes it easier to study the figures of a single company over a period of time, or to compare the figures of one company with those of another company or group of companies. In Fig. 53, the December 31 figures of the X. Y. Z. Company (in thousands of dollars) are shown over a period of four years. In converting the actual figures into percentages, we may use any year in the series as a base.

If the first year, 1940, is selected, then the amounts in the other years are expressed as percentages of the 1940 amounts by dividing them by the 1940

	1940	1941	1942	1943
Current assets	\$ 7,036	\$ 9,959	\$ 9,095	\$ 8,123
Fixed assets	13,671	12,838	11,403	12,402
Deferred charges	128	237	229	155
Current liabilities	2,341	3,912	3,481	3,162
Capital stock	12,612	12,607	12,612	13,412
Surplus	5,882	6,515	4,634	4,106

Fig. 53

figures; *e.g.*, the percentage of current assets for the year, 1941, is calculated by dividing \$9,959 by 7,036; the current assets percentage for 1942 is calcu-

	1940	1941	1942	1943
Current assets	100.0%	141.5%	129.3%	115.4%
Fixed assets	100.0	93.9	83.4	90.7
Deferred charges	100.0	185.2	178.9	121.1
Current liabilities	100.0	167.1	148.7	135.1
Capital stock	100.0	100.0	100.0	106.3
Surplus	100.0	110.8	78.8	69.8

Fig. 54

lated by dividing \$9,095 by 7,036; etc. The dollar amounts, reduced to percentage form, are shown in Fig. 54.

Certain significant trends and relationships among the balance-sheet accounts of the X. Y. Z. Company are now evident. Both current assets and current liabilities increased the second year and then declined in the third and fourth years, but by 1943 the current liabilities were 35.1% greater than in the base year whereas the current assets were only 15.4% greater. The fixed assets declined 9.3% over the same period, probably due to depreciation. Both fixed assets and capital stock increased slightly from 1942 to 1943. This probably means an expansion in the business, financed by the sale of stock. There is a noticeable decline in surplus, caused possibly by operating losses or dividend payments or both.

TEST YOUR KNOWLEDGE OF TREND-PERCENTAGES BY THESE PROBLEMS

- 14 The dollar sales of three companies in the same industry appear below. Convert the actual figures into trend-percentages, using the first year as a base. Which company makes the best showing over the five-year period?

	1939	1940	1941	1942	1943
Company J	\$ 18,516,740	\$ 20,397,720	\$ 27,075,890	\$ 31,949,205	\$ 30,281,110
Company K	180,358,903	231,506,460	204,711,210	242,281,600	230,604,412
Company L	145,207	183,612	210,210	200,460	218,192

- 15 Using the year, 1941, as a base, recalculate the trend-percentages of the figures in the preceding question. Notice the effect on the percentages of the change in the base year.
- 16 A certain business in applying for a loan was required to submit its detailed current assets over the past four years. The figures were as follows:

	1940	1941	1942	1943
Cash	\$35,773	\$32,481	\$33,220	\$30,461
Marketable securities	26,916	27,112	33,407	20,660
Receivables—net	10,776	11,500	8,352	9,610
Inventories	23,388	20,109	27,614	35,886

Convert these figures into trend-percentages, using 1940 as the base. What conclusions might be drawn from this analysis?

Ratio method of analysis

Ratios may be used for analyzing either single statements or several statements. Before undertaking such analysis, we customarily arrange the accounts in comparable groups or in logical sequence, and, if necessary, simplify the figures. The balance sheet of the D. D. B. Company, as of December 31, 1943 (Fig. 49), illustrates the proper method of preparing a single statement for ratio analysis. Numerous relationships can be studied but the following ratios, calculated from the figures of the D. D. B. Company, are typical:

$$\begin{aligned}\frac{\text{current assets}}{\text{current liabilities}} &= \frac{\$ 3,972,000}{\$ 1,377,000} = 288.5\% \\ \frac{\text{current assets} - \text{inventories}}{\text{current liabilities}} &= \frac{\$ 1,846,000}{\$ 1,337,000} = 134.1\% \\ \frac{\text{current assets}}{\text{total assets}} &= \frac{\$ 3,972,000}{\$ 20,488,000} = 19.4\% \\ \frac{\text{net worth}}{\text{fixed assets}} &= \frac{\$ 14,101,000}{\$ 15,940,000} = 87.9\% \\ \frac{\text{net worth}}{\text{total liabilities}} &= \frac{\$ 14,101,000}{\$ 6,387,000} = 220.8\%\end{aligned}$$

The purpose of the first of these ratios, frequently referred to as the *current* ratio, is to test the current solvency, or debt-paying ability, of a business. As inventories of merchandise must be sold and converted into cash before they can be used for paying debts, a more severe test of a company's debt-paying ability is found in the second ratio, referred to sometimes as the *acid test*.

The current ratio of 288.5% for the D. D. B. Company means that it has \$2.88 of current assets for every dollar of current liabilities. The acid test of 134.1% means that the company still has \$1.34 of current assets for every dollar of current liabilities after eliminating the merchandise inventories from the current asset total.

The ratio of current assets to total assets shows merely the proportion of a company's assets which are in current or liquid form.

In the case cited, approximately one-fifth of the assets are current and four-fifths are either fixed or deferred.

In some businesses, there should be a definite relationship between the capital provided by the owners (shown as capital stock and surplus in the case of a corporation) and the fixed, or non-current, assets. If the net worth is greater than the fixed assets, one will probably find that the owners have provided both the permanent capital and a portion of the current, or *working*, capital.

In the illustration under consideration, the net worth is only 87.9% of the fixed assets. This means, apparently, that some of the permanent financing has been done through the issuing of bonds or other forms of indebtedness.

The ratio of net worth to total liabilities emphasizes the relationship between the capital provided by the owners (capital stock and surplus)

and that provided by the creditors (fixed and current liabilities).

A ratio of 220.8%, in the illustration above, means that the owners have supplied about $2\frac{1}{4}$ times as much capital as the creditors.

The ratio method of analysis is used also in connection with income statements. Typical operating ratios, calculated from the 1942 earnings statement of the X. Y. Z. Company (Fig. 51), are as follows:

$$\begin{aligned}\frac{\text{operating expenses}}{\text{net sales}} &= \frac{\$17,933,000}{\$20,398,000} = 87.9\% \\ \frac{\text{net earnings}}{\text{net sales}} &= \frac{\$1,738,000}{\$20,398,000} = 8.5\% \\ \frac{\text{amount available for common dividends}}{\text{net sales}} &= \frac{\$1,484,000}{\$20,398,000} = 7.3\%\end{aligned}$$

The reader will note that all of these operating ratios are disclosed by the percentage statement of the X. Y. Z. Company. The first one, operating expenses to net sales, is not shown as a separate figure but it can be found by adding the cost of goods sold percentage (64.1) to the distribution, administrative, and general expenses percentage (23.8). The first ratio is a test of the management's ability to keep operating expenses within operating income, whereas the second and third ratios are measures of the earning power of the business.

INTERSTATEMENT OR TURNOVER RATIOS

When both the balance sheet and the income statement of a company are analyzed, *interstatement* or *turnover* ratios are employed. These ratios make use of figures on both reports and show significant relationships which the separate analysis of either statement would not disclose.

By referring again to the December 31, 1943, balance sheet of the D. D. B. Company and by assuming net sales for the year of \$7,320,000, we can develop the following typical turnover ratios:

$$\begin{aligned}\frac{\text{net sales}}{\text{receivables}} &= \frac{\$7,320,000}{\$1,371,000} = 433.9\% & \frac{\text{net sales}}{\text{fixed assets}} &= \frac{\$7,320,000}{\$15,940,000} = 45.9\% \\ \frac{\text{net sales}}{\text{inventories}} &= \frac{\$7,320,000}{\$2,126,000} = 344.3\% & \frac{\text{net sales}}{\text{net worth}} &= \frac{\$7,320,000}{\$14,101,000} = 51.9\%\end{aligned}$$

The reason for the net sales to receivables ratio is to test the collection policy of the business. If a company plans to collect its receivables every 30 days, its annual sales should be approximately 12 times the amount of the receivables which are outstanding at any particular time. A ratio of 1200%, therefore, means collections in 30 days; 2400% means collections twice as often, or every 15 days; etc. The following formula, based on a 360-day year, is useful in converting the net sales to receivables ratio into the number of days in which collections are made:

$$360 \div \text{ratio of net sales to receivables} = \text{number of days. XXXIII}$$

Substituting the figures of the D. D. B. Company in XXXIII, we have:

$$360 \div 433.9\% \text{ (or } 4.4) = 82 \text{ days.}$$

In interpreting this ratio, we must keep in mind several modifying factors. If the actual receivables at the end of the fiscal period are either larger or smaller than the average receivables during the year, the average collection policy will not be disclosed. Also, as the net sales figure usually includes both sales for cash and sales on account, this figure is not properly comparable with receivables, which can only result from credit sales. A more accurate method of testing the collection policy is to divide the credit sales by the average monthly receivables.

The purpose of the net sales to inventories ratio is to test the turnover of the merchandise; in other words, to determine how many times per period the average stock of merchandise is sold. If the fiscal period is a year in length, a ratio of 200% would mean a turnover of twice a year; 600% would mean 6 times; 1200% would mean 12 times, or every 30 days; etc.

In the illustration, the ratio is 344.3%, which means a turnover of approximately $3\frac{1}{2}$ times a year.

However, there are likewise modifying factors in connection with this ratio. As sales are usually valued at a higher price than the merchandise inventories, the two figures are really not comparable. Also, the inventory figure at the end of the fiscal period may not correctly reflect the average inventory throughout the year. A more accurate measure of merchandise turnover is found by dividing the cost of goods sold by the average monthly merchandise inventory.

The purpose of the last two ratios—net sales to fixed assets, and net sales to net worth—is to determine the earning capacity of the fixed assets and of the owners' investment in terms of volume of business. Usually the fixed assets mean the plant assets, or those actually used in production, and do not include intangible assets, investments in the securities of other companies, nor deferred charges. As the net worth appearing on the balance sheet at the end of a fiscal period usually includes the profit resulting from the sales of the period, a better measure of the earning power of the investors' capital is obtained by dividing the net sales for the period by the net worth at the beginning of the same fiscal period, or by the average investment of the owners throughout the period.

COMPARATIVE OR STANDARD RATIOS

As it is difficult to know whether a ratio for a particular company is good, bad, or indifferent, *comparative* or *standard* ratios are the next logical development in the ratio method of analysis. An *internal* standard ratio is developed by finding the average ratio of a company over a number of periods and an *external* standard ratio is developed by calculating the average ratio of a number of companies for the same

fiscal period. The first, or internal, type can be illustrated by referring to the December 31 figures of the X. Y. Z. Company, as follows:

Ratio of Current Assets to Current Liabilities		
1940	$\frac{\$7,036}{\$2,341}$	= 300.6%
1941	$\frac{\$9,959}{\$3,912}$	= 254.6%
1942	$\frac{\$9,095}{\$3,481}$	= 261.3%
1943	$\frac{\$8,123}{\$3,126}$	= 259.9%
	$\frac{4)1076.4}{269.1}$	= internal standard ratio

This type of ratio is useful in showing a business whether the results of a particular year are above or below the average; it does not show a concern how it stands in relation to other companies in the same industry.

If it is assumed that the 27 figures listed on page 1059 represent the current ratios (expressed as whole numbers) of 27 companies in a certain industry, the external standard ratio would be 454% or 453%, depending on whether the arithmetic mean or the median or the mode were used as the average. If this is the standard ratio for the year, 1943, and the X. Y. Z. Company shows a ratio of 259.9% for the same year, it is evident that it is 284% below the average.

Instead of obtaining the arithmetic mean by merely averaging the current ratios, we might divide the aggregate of the current assets of the 27 companies by the aggregate of the current liabilities, provided such detailed information is available.

A further development of external standard ratios is explained in the next section on business indices.

TEST YOUR KNOWLEDGE OF THE RATIO METHOD OF ANALYSIS

- 17 From the figures of the Jones Corporation given in Fig. 55, calculate the twelve balance-sheet, income-statement, and turnover ratios referred to in the preceding section.

Business-index method of analysis

As it is difficult to judge the condition of a company from a set of conflicting ratios, some of which appear to be favorable and others unfavorable, a *business-index* method of analysis has been devised to summarize the ratios and interpret them through a single figure. This figure is supposed to indicate in a very general way how a particular concern compares with a group of other companies in the same industry.

As it is commonly recognized that certain ratios are more significant than others, an attempt is made in this method to *weigh* or decide the relative importance of each ratio before including it in the analysis. One analyst, for example, may decide that the current ratio is twice

as important as either the net worth to fixed assets or the net worth to total liabilities ratio. If these are the only three ratios involved in an analysis, their weights would be 50%, 25%, and 25%, respectively.

Balance Sheet as of December 31, 1943

Current assets:			Current liabilities			\$ 8,824
Cash	\$ 1,451		Long-term debt			<u>4,142</u>
Receivables	6,210		Total liabilities			\$12,966
Inventories	<u>10,034</u>	\$17,695	Net worth:			
Fixed assets less			Capital stock	\$3,000		
reserves for			Surplus	<u>7,962</u>	10,962	
depreciation	6,078					
Deferred charges	155		Total liabilities			
Total assets	<u>\$23,928</u>		and net worth			<u>\$23,928</u>

Income Statement for the year ending December 31, 1943

Net sales	\$25,581
Cost of goods sold	<u>20,011</u>
Gross profit	\$ 5,570
Other operating expenses	<u>3,118</u>
Net profit from operations	\$ 2,454
Other income	<u>420</u>
	\$ 2,874
Other expenses and losses	<u>812</u>
Net profit available for dividends	\$ 2,062
Dividends declared and paid	<u>210</u>
Balance to surplus	<u>\$ 1,852</u>

Fig. 55

Unfortunately, analysts are not agreed on the relative importance of various ratios; the weights, therefore, depend on the judgment of the individual making the analysis and on the particular ratios involved.

In this particular method of analysis, each ratio of the company being studied is divided by the standard ratio of the industry in order to form a new ratio, thus:

$$1943 \text{ current ratio: } \frac{\text{X. Y. Z. Company}}{\text{industry}} = \frac{260\%}{454\%} = 57.3\%.$$

The resulting ratio is then multiplied by the weight assigned; e.g., if the current ratio is given a weight of 50%, then $50\% \times 57.3\% = 28.65\%$ (or 28.65). As a company-to-industry ratio of 100% indicates equality between the company and the industry as a whole, the 57.3% in this case means that the X. Y. Z. Company's current position is 42.7% below the average. This sub-standard position is further reflected in the 28.65 figure (which would be 50.00 with a company-to-industry ratio of 100%). Each ratio included in a particular analysis is handled in the same manner and the sum of the figures thus obtained is the index of the business. If the index is less than 100, it means that the company is below the average of the industry; on the other hand, if the index exceeds 100, it means that the company is above the average. The following figures illustrate an index of this type:

TYPE OF RATIO	COMPANY RATIO	÷	INDUSTRY RATIO	=	RATIO OF COMPANY TO INDUSTRY	×	WEIGHT	=	INDEX
<u>current assets</u>	420%	÷	460%	=	91.3%	×	30%	=	27.39
<u>current liabilities</u>									
<u>net worth</u>	210	÷	300	=	70.0	×	30	=	21.00
<u>total liabilities</u>									
<u>net sales</u>	1220	÷	1480	=	82.4	×	10	=	8.24
<u>receivables</u>									
<u>net sales</u>	680	÷	700	=	97.1	×	10	=	9.71
<u>inventories</u>									
<u>net sales</u>	240	÷	510	=	47.1	×	20	=	9.42
<u>net worth</u>							<u>100%</u>		
					Index.....				<u>75.76</u>

In this method of analysis, a company ratio may be either greater or less than the corresponding standard ratio of the industry. A problem arises if a company ratio is greatly in excess of the standard because such a condition may distort the final index. It is impossible for a company ratio to be more than 100% *below* standard, but mathematically it is possible for it to be 100%, 200%, 1000%, or any amount *above* standard. In order to prevent the final index from being completely upset by one or more extreme ratios, a formula is sometimes used if the company ratio is higher than the industry ratio. By this device, the variations from the standard are arbitrarily limited to 100% in either direction. The formula is expressed:

$$200\% - \frac{\text{industry ratio}}{\text{company ratio}} = (\text{controlled}) \text{ ratio of company to industry. XXXIV}$$

If it is assumed in a certain case that a company ratio is 800% and the corresponding standard ratio is 200%, the formula would give a resulting ratio of 175%, as shown below:

$$200\% - \frac{200\%}{800\%} = 200\% - 25\% = 175\%.$$

Without the use of this formula, this ratio would be:

$$\frac{\text{company ratio}}{\text{industry ratio}} = \frac{800\%}{200\%} = 400\%.$$

In the next illustration, the formula is used in connection with the last two ratios; the other ratios are calculated in the usual manner.

TYPE OF RATIO	COMPANY RATIO	÷	INDUSTRY RATIO	=	RATIO OF COMPANY TO INDUSTRY	×	WEIGHT	=	INDEX
<u>current assets</u>	480%	÷	500%	=	96.0%	×	40%	=	38.40
<u>current liabilities</u>									
<u>current assets—inventory</u>	120	÷	150	=	80.0	×	10	=	8.00
<u>current liabilities</u>									
<u>net worth</u>	440	÷	400	=	109.1	×	30	=	32.73
<u>total liabilities</u>									
<u>net worth</u>	630	÷	215	=	165.9	×	20	=	33.18
<u>fixed assets</u>							<u>100%</u>		
					Index.....				<u>112.31</u>

TEST YOUR KNOWLEDGE OF THE BUSINESS-INDEX METHOD OF ANALYSIS

18 From the ratios of the Brown Company and of the industry of which it is a part, together with the suggested weights, calculate a business index.

RATIO	BROWN COMPANY	INDUSTRY	WEIGHT
Current	450%	330%	30%
Sales to receivables	1040	1112	15
Sales to inventories	560	640	15
Net worth to total liabilities	160	320	25
Net profit to net worth	17	11	15

19 Reverse the weights in the preceding question, beginning with 15% for the current ratio and ending with 30% for the net profit to net worth ratio, and recalculate the index. Notice the effect of the change in weights on the index.

Sources and application of funds method of analysis

Another method of analyzing accounting statements is by sources and application of funds. The word, *fund*, as used in this connection, means either cash or net working capital (current assets minus current liabilities); it does not refer to sinking funds or sinking-fund assets which sometimes appear on the balance sheet of a concern.

The purpose of this method is to determine why and how working capital has increased or decreased during a period; it is, therefore, primarily an analysis of changes in the financial structure of the business. Current funds arise from increases in fixed liabilities (long-term borrowing), increases in proprietorship (investments and earnings of owners), and decreases in non-current assets (by conversion) whereas funds are applied to the payment of fixed liabilities, the reduction of proprietorship (for example, the payment of dividends), and the increase of non-current assets (acquisition). These changes can be expressed in equation form, as follows:

net changes in assets = $\frac{\text{net changes in liabilities}}{\text{and proprietorship}}$ XXXV

$$\left. \begin{array}{l} \text{increase in assets} \\ \text{decrease in liabilities} \\ \text{decrease in proprietorship} \end{array} \right\} = \left\{ \begin{array}{l} \text{increase in liabilities} \\ \text{increase in proprietorship} \\ \text{decrease in assets} \end{array} \right. \text{ XXXVI}$$

application of funds = sources of funds XXXVII

This method of analysis involves three distinct steps:

- a The calculation of increases and decreases of assets, liabilities, and proprietorship from comparative balance sheets,
- b The determination of the net change in working capital, and
- c The preparation of a statement of sources and application of funds.

These steps are illustrated in Fig. 56.

In preparing a statement of sources and application of funds, we must add back to the net income figure any expenses or losses which have come out of profits but which have required no outlay of funds (for example, depreciation and loss on the disposal of fixed assets).

S. AND A. COMPANY

Comparative Condensed Balance Sheets as of December 31

(Figures in thousands of dollars)

ASSETS	1942	1943	Increase	Decrease
Current (net)	\$12,514	\$11,159		\$1,355
Fixed (net)	8,372	8,599	\$227	
Intangibles	372	341		31
Deferred charges	138	184	46	
Total assets	<u>\$21,396</u>	<u>\$20,283</u>	<u>\$273</u>	<u>\$1,386</u>
LIABILITIES AND NET WORTH				
Current liabilities	\$ 2,246	\$ 1,722		\$ 524
Fixed liabilities	980	720		260
Capital stock	12,013	12,013		
Surplus	6,157	5,828		329
Total liabilities and net worth	<u>\$21,396</u>	<u>\$20,283</u>		<u>\$1,113</u>

Surplus Statement for the year ending December 31, 1943

Balance of surplus, 12/31/42		\$6,157
Add: Net income for the year 1943	\$841	
Refund of taxes for prior years	<u>30</u>	871
		\$7,028
Less: Dividends declared and paid		1,200
Balance of surplus, 12/31/43		<u>\$5,828</u>

Change in Working Capital between December 31, 1942, and December 31, 1943

Decrease in current assets	\$1,355
Decrease in current liabilities	524
Net decrease in working capital	<u>\$ 831</u>

Sources of Funds

Net income for the year 1943	\$ 841
Add back: Expenses and losses not requiring an outlay of funds:	
Depreciation of fixed assets	<u>335</u>
Total funds from operations	<u>\$1,176</u>
Add: Funds from other sources:	
Refund of taxes of prior years	\$30
Decrease in intangible assets	<u>31</u>
Total funds from all sources	<u>\$1,237</u>

Application of Funds

Payment of fixed liabilities	\$ 260
Payment of dividends	1,200
Acquisition of fixed assets	582
Acquisition of deferred charges	<u>46</u>
Total funds applied	<u>2,068</u>
Excess of funds applied over funds raised	<u>\$ 831</u>

Fig. 56

Only by doing this is it possible to show the total funds which have resulted from current operation of the business.

In the illustration above, the net increase in the fixed assets is accounted for as in Fig. 57.

In interpreting the statement of sources and application of funds shown in Fig. 56, we find it apparent that the funds arising from operations (\$1,176) were insufficient to meet the dividend requirements (\$1,200). The retirement of the fixed liabilities (\$260) and the acquisition of additional fixed assets and deferred charges (\$608) were made possible by a further reduction in working capital.

Balance, 12/31/42	\$8,372
Less: Depreciation	335
	<u>\$8,037</u>
Add: New assets acquired	562
Balance, 12/31/43	<u>\$8,599</u>

Fig. 57

TEST YOUR KNOWLEDGE OF SOURCES AND APPLICATION OF FUNDS

- 20 Which of the following transactions would give rise to funds, which are applications of funds, and which are neither sources nor applications?

(a) The declaration and payment of a cash dividend. (b) The declaration and payment of a stock dividend. (c) A fire loss which was not entirely covered by insurance. (d) The writing off of an intangible asset directly against surplus. (e) The creation of a reserve for contingencies by a charge against surplus. (f) The sale of preferred stock. (g) Depreciation of fixed assets.

STONE CORPORATION

ASSETS	12/31/42	12/31/43
Current assets (net)	\$ 52,113	\$ 49,277
Fixed assets (net)	80,361	79,343
Intangibles	1	1
Deferred charges	1,069	667
Total assets	<u>\$133,544</u>	<u>\$129,288</u>

LIABILITIES AND NET WORTH		
Current liabilities	\$ 12,900	\$ 8,105
Fixed liabilities	2,000	4,000
Capital stock	116,739	115,392
Surplus	1,905	1,791
Total	<u>\$133,544</u>	<u>\$129,288</u>

- 21 From the comparative balance sheets and supplementary information of the Stone Corporation, given in Fig. 58, prepare a statement of sources and application of funds.

Net income for the year, \$3,800
Dividends paid, \$3,914
Depreciation, \$3,200

Fig. 58

- 22 From the following information, prepare a statement of sources and application of funds: depreciation charged off during the year on buildings, \$6,000; on machinery, \$28,500. New machinery was purchased for \$30,000, and \$12,000 credit on the new machinery purchased was allowed through trading in old machinery with a book value at the time of trade-in of \$22,500. The balance of the purchase was paid in cash. During the year, patents were written off to the amount of \$2,000, and \$5,000 in new patents were added. Holders of first-mortgage bonds, amounting to \$25,000, exchanged their bonds at par with the corporation for capital stock at par. In addition, capital stock amounting to \$11,000 was sold at par for cash and \$50,000 of bonds were paid off by the corporation at face value. The surplus statement of the corporation showed: a balance at the beginning of the year of \$222,000; a net profit for the year of \$42,000; deductions for the loss of a damage suit, \$25,500; organization expenses written off, \$30,000; and a cash dividend declared and paid off, \$30,000; leaving a balance at the end of the year in surplus of \$205,500. Working capital decreased \$28,500 during the year.

• SPECIALIZED ACCOUNTING PROBLEMS •

By Jeremiah Lockwood, A.M., C.P.A.

MANY problems besides those previously mentioned require the use of mathematics by the accountant in carrying on his activities. Some of these problems are very technical, of general interest and extensive application. Many of the problems in the field of cost accounting are of this nature. The behavior of fixed and variable costs must be understood; wage-payment incentive and premium plans must be worked out; the effect on depreciation charges of speeding up machine production and of operating machines for long periods of time without rest or repairs must not be overlooked; the apportionment of material, labor, and overhead or burden costs must be carefully made if reasonably accurate costs of manufactured articles are to be obtained.

There are other problems in accountancy which are highly specialized and of more limited application which require the use of mathematics in their solutions. The apportionment of profits in a building-and-loan association, the determination of inventory values for department stores, the interdependence of State and federal taxes, and of taxes and executive bonuses are illustrative of this type of problem which is limited in application. The voluminous literature on accounting carries numerous references to mathematical formulae which may be useful to the accountant in solving problems of accounting whether of general application or of limited use.

It is, of course, impossible to discuss all of the additional problems encountered by business men and requiring the use of mathematical formulae. Although many of them may be solved by arithmetic, the process of solving them may be shortened considerably if mathematical equations are used. Whether acting as business advisor, as an auditor, or as an income tax specialist, an accountant will find that a knowledge of and an ability to apply mathematical formulae or principles to the solution of business problems are indispensable. This article will indicate how mathematics may be utilized in analyzing profit-and-loss statements, in preparing break-even charts, in determining executives' bonuses, and in solving problems relating to the interdependence of State and federal taxes.

**PROFIT-AND-LOSS
ANALYSES**

A business man frequently finds it necessary to secure an analysis of the operations of his enterprise which is more enlightening than one based on percentages, ratios, or trends. A superficial ratio analysis may disclose changes in the sales or net profits but is of little value in determining the reasons for the change. For example, assume that the profit-and-loss statements of the Walnut Manufacturing Company (Fig. 59) have been prepared for the years 1943 and 1944. (Figures have

been summarized for purposes of this and subsequent illustrations.)

The condensed statements indicate that there is an increase of \$1,000 in net profit in 1944, as compared with 1943. The percentages do not

WALNUT MANUFACTURING COMPANY

	—1943—		—1944—		Increase or Decrease
	Amount	%	Amount	%	
Sales	\$300,000	100	\$400,000	100	\$100,000
Cost of goods sold	210,000	70	288,000	72	78,000
Gross profit	\$ 90,000	30	\$112,000	28	\$ 22,000
Selling and administrative expenses	87,000	29	108,000	27	21,000
Net profit	<u>\$ 3,000</u>	<u>1</u>	<u>\$ 4,000</u>	<u>1</u>	<u>\$ 1,000</u>

Fig. 59

indicate the reasons for this increase. In preparing a more detailed analysis, we should state that increases or decreases in business profits are due to

- a Price level changes,
- b Volume variances,
- c Managerial efficiency or inefficiency,
- d Combinations of these factors.

A change in unit selling prices may become necessary due to price-cutting activities of competitors. A failure to act realistically often results in a merchant or manufacturer's losing his market and in accumulating stock which cannot be sold at prices above those of competitors. The inventory of merchandise may become obsolete and eventually may have to be sold at sacrifice prices to prevent greater losses. In a period of inflation, selling prices may be arbitrarily adjusted upward to take advantage of the current up-swing. There is no direct relationship between managerial efficiency and profits in a period of rising prices or falling prices. A very efficient business man may lose money during the period of price deflation and an inefficient and marginal manufacturer or merchant may make large profits during a period of price inflation.

Variances in volume of merchandise sold will bring about changes in net profits. Increased volume should increase net profits; decreased volume results as a rule in decreased net profits. This holds true whether prices are stabilized, increasing, or decreasing.

Other things being equal, managerial efficiency will be reflected in net profits. Failure to maintain efficiency in manufacturing or sales activities will result in lowered profits.

Shipping records and customer billing will, as a rule, enable one to determine the volume or tonnage handled, especially when the business unit is specializing in the manufacture of one or two products. Cement manufacturers, producers of special types of paints, brick manufacturers, ice manufacturers, or producers of electricity will have

little difficulty in determining changes in volume from one period to another.

Let us refer to the previous illustration (The Walnut Manufacturing Company) and proceed to analyze the profit-and-loss data on the basis of the following assumptions:

- a Assume an increase in the average sales price of 25% per unit in 1944 as compared with 1943.

Since the figures for 1943 are to be used as a basis or guide in making the

	1943 VOLUME		1944 VOLUME ADJUSTED FOR PRICE CHANGE		1944 ACTUAL	Analysis of Net Profit Changes		
	Amount	%	Amount	%		1944 CHANGES IN VOLUME AS COMPARED WITH 1943	CHANGES IN SELLING PRICE	CHANGES IN COST OF PRODUCTION AND IN SELL- ING COST
Sales	\$300,000	100	\$320,000	100	\$400,000	\$20,000	+\$80,000	—
Cost of goods sold	210,000	70	224,000	70	288,000	14,000	—	\$64,000
Gross profit	\$ 90,000	30	\$ 96,000	30	\$112,000	\$ 6,000	+\$80,000	—\$64,000
Selling and administrative expenses	87,000	29	92,800	29	108,000	5,800	—	15,200
Net profit	\$ 3,000	1	\$ 3,200	1	\$ 4,000	+\$ 200	+\$80,000	—\$79,200
Changes in net profits as compared with 1943						Increase		\$ 1,000

Fig. 60

analysis, it becomes necessary to adjust 1944 data to 1943. The comparable sales figure for 1944 can be secured by dividing \$400,000 by 125%, the new price prevailing. This results in a total sales dollar volume of \$320,000. A statement may then be prepared (as in Fig. 60) to indicate the changes due to changes in price and in volume.

The analysis reveals that the increase of \$1000.00 in net profit is due to a profit increase of \$200.00 because of volume changes, a profit increase of \$80,000.00 because of changes in selling price, and a profit decrease of \$79,200.00 because of changes in cost of goods sold and operating expenses.

	1943 VOLUME		1944 ADJUSTED FOR VOLUME CHANGE OF 20% INCREASE		1944 ACTUAL	Analysis of Net Profit Changes		
	Amount	%	Amount	%		1944 CHANGES IN VOLUME AS COMPARED WITH 1943	CHANGES IN SELLING PRICE	CHANGES IN COST OF PRODUCTION AND IN SELL- ING COST
Sales	\$300,000	100	\$360,000	100	\$400,000	\$60,000	\$40,000	—
Cost of goods sold	210,000	70	252,000	70	288,000	42,000	—	36,000
Gross profit	\$ 90,000	30	\$108,000	30	\$112,000	\$18,000	+\$40,000	—\$36,000
Selling and administrative expenses	87,000	29	104,400	29	108,000	17,400	—	3,600
Net profit	\$ 3,000	1	\$ 3,600	1	\$ 4,000	+\$ 600	+\$40,000	—\$39,600
Changes in net profits as compared with 1943						Increase		\$ 1,000

Fig. 61

- b Assume an increase of 20% in volume of merchandise sold in 1944 as compared with 1943.

The analysis is made in a manner similar to a. The 1943 figures should be adjusted for the increase in volume and compared with 1944. A statement may then be prepared (as in Fig. 61) to indicate the changes due to changes in price and in volume.

The analysis reveals that the increase of \$1000.00 in net profit is due to a

profit increase of \$600.00 because of volume changes, a profit increase of \$40,000.00 because of changes in selling price, and a profit decrease of \$39,600.00 because of changes in the cost of goods and operating expenses.

The analysis may be carried further if one is desirous of knowing what specific price changes, if any, are responsible for the increases in the cost of goods sold and selling and administrative expenses in 1944 as compared with 1943. If price factors have not entered into the increases in these items, management is responsible and should be charged with the resultant increases in expense because of its failure to maintain a level of efficiency as high as during the preceding year.

TEST YOUR KNOWLEDGE OF PROFIT-AND-LOSS ANALYSIS

The following comparative profit-and-loss statements are presented to you for analysis:

	1943	1944
Sales	\$60,000	\$70,000
Cost of sales	40,000	54,000
Gross profit	<u>\$20,000</u>	<u>\$16,000</u>
Selling and administrative expenses	16,000	18,000
Net profit (+) or loss (-)	<u>+\$ 4,000</u>	<u>-\$ 2,000</u>

- 1 Assume that the number of units sold in 1944 was 30% in excess of the number sold in 1943. Prepare the desired analysis.
- 2 Assume that the sales price of each unit sold was \$2.00 in 1943 and \$3.50 in 1944. Prepare the desired analysis.
- 3 Assume that the company sold 20,000 units in 1943 and 18,500 in 1944. Prepare the desired analysis.
- 4 Assume that the company sold each unit for \$3.00 in 1943 and for \$2.80 in 1944. Prepare the analysis of profit-and-loss changes.

Volume, price, and profit relationships

We have just noted that it is possible to analyze a profit-and-loss statement and determine the effect on net profit of a change in volume or a change in the unit price of product sold. In making business decisions and in formulating policies, we often find it necessary and desirable to project the operating statement so as to reflect the results of contemplated decisions. A company president may ask the accountant for a statement showing the effect of an increase or decrease in volume or an increase or decrease in price or an increase in one factor offset by a decrease in the other factor.

An illustration will be of value in making this clear. Instead of arranging the profit-and-loss data in the form of sales, cost of sales, gross profit, selling and administrative expenses, and net profit, we shall develop a slightly different presentation. Expenses or costs will be classified as fixed or variable. The fixed costs are those items of cost which remain fairly constant even though the volume of production fluctuates. Rent must be paid regardless of the units produced; property taxes do not fluctuate with volume of output; minimum insurance premiums are incurred even though the plant is almost

idle; depreciation charges on machinery and buildings continue although the plant may be shut down. These are illustrations of fixed costs or charges. Variable costs fluctuate directly and almost proportionately with production. Variable costs at 25% of capacity may total \$4,000. At 50%, they will tend to approximate \$8,000; at 75%, \$12,000; and so on. When the plant is idle, variable expenses or costs are practically non-existent.

Illustrative Problem

The Pyrex Heater Company manufactures fireplace grates. The president of the company has requested the accounting department to prepare statements showing the effect on profits of a 10% increase in volume, a 10% decrease in volume, a 10% increase in unit sales prices,

Sales (40,000 units @ \$10 per unit)	\$400,000
Less: Cost of sales including selling and administrative expenses	
Fixed costs	\$106,000
Variable costs (\$5.45 per unit)	218,000
Total costs	<u>324,000</u>
Net profit	<u>\$ 76,000</u>
Net profit per unit	<u>\$1.90</u>

(a) Influence of Volume Changes on Net Profits

	VOLUME 10% BELOW NORMAL (or 36,000 Units)	VOLUME NORMAL (or 40,000 Units)	VOLUME 10% ABOVE NORMAL (or 44,000 Units)
Sales (\$10 per unit)	\$360,000	\$400,000	\$440,000
Less: Cost of sales			
Fixed costs	\$106,000	\$106,000	\$106,000
Variable costs (at \$5.45 per unit)	196,200	218,000	239,800
Total costs	<u>\$302,200</u>	<u>\$324,000</u>	<u>\$345,800</u>
Net profit	<u>\$ 57,800</u>	<u>\$ 76,000</u>	<u>\$ 94,200</u>
Net profit per unit	<u>\$ 1.605</u>	<u>\$ 1.90</u>	<u>\$ 2.121</u>
% change in net profit	-23.95%	—	+23.95%
% change in volume	-10%	—	+10%

(b) Influence of Price Changes Upon Net Profits

	PRICE 10% BELOW NORMAL (or \$9.00 Per Unit)	PRICE AT NORMAL (or \$10.00 Per Unit)	PRICE 10% ABOVE NORMAL (or \$11.00 Per Unit)
Sales (40,000 units)	\$360,000	\$400,000	\$440,000
Less: Cost of sales			
Fixed costs	\$106,000	\$106,000	\$106,000
Variable costs (at \$5.45 per unit)	218,000	218,000	218,000
Total costs	<u>\$324,000</u>	<u>\$324,000</u>	<u>\$324,000</u>
Net profit	<u>\$ 36,000</u>	<u>\$ 76,000</u>	<u>\$116,000</u>
Net profit per unit	<u>\$ 0.90</u>	<u>\$ 1.90</u>	<u>\$ 2.90</u>
% change in net profit	-52.6%	—	+52.6%
% change in price	-10%	—	+10%

(c) Influence of Both Volume Changes and Price Changes on Net Profits

	-10% PRICE +10% VOLUME (44,000 @ \$9.00 Per Unit)	NORMAL (40,000 @ \$10.00 Per Unit)	+10% PRICE -10% VOLUME (36,000 @ \$11.00 Per Unit)
Sales	\$396,000	\$400,000	\$396,000
Less: Cost of sales			
Fixed costs	\$106,000	\$106,000	\$106,000
Variable costs (at \$5.45 per unit)	239,800	218,000	196,200
Total costs	<u>\$345,800</u>	<u>\$324,000</u>	<u>\$302,200</u>
Net profit	<u>\$ 50,200</u>	<u>\$ 76,000</u>	<u>\$ 93,800</u>
Net profit per unit	<u>\$ 1.141</u>	<u>\$ 1.90</u>	<u>\$ 2.60</u>
% change in net profit	-34%	—	+23.4%

and a 10% decrease in unit sales prices. The profit-and-loss statement on page 1075, greatly condensed, shows the sales, total cost of sales, and net profits based on normal operations of 40,000 units per year.

We may generalize from this. A percentage change in selling prices has a *greater* effect upon net profits than the same percentage change in volume of units sold. This is particularly noticeable in the preceding example. An increase of 10% in volume accompanied by a 10% decrease in unit price results in a considerable reduction of profit, whereas an increase of 10% in price accompanied by a 10% decrease in volume results in an increase in profits.

TEST YOUR KNOWLEDGE OF VOLUME, PROFIT, AND PRICE RELATIONSHIPS

- 5 Would it be to the advantage of the Pyrex Heater Company to cut unit prices 10% if by so doing an increase of 20% in volume could be secured? Make calculations supporting your conclusions.
- 6 By re-arranging plant machinery and employing a wage incentive plan, we believe that variable costs of the Pyrex Company can be reduced to \$5.00 per unit. This saving can be effected only if 50,000 units can be produced and sold. In order that all units may be sold, we shall find it necessary to reduce the sales price per unit to \$9.25. Would you suggest that plant machinery be re-arranged and a wage-incentive plan placed in operation? Prepare calculations to prove your contention.
- 7 Which is more advantageous: to reduce unit prices 5% and increase volume 10%, or to increase unit prices 5% if such an increase will result in a volume reduction of 10%?
- 8 Use the data in the illustration on page 1075 for normal production of 40,000 units and normal sales price of \$10.00 per unit. The president of the company would like to know the number of units he would have to sell if prices were reduced 5% and a net profit of approximately \$60,000 were to be secured.

The break-even point

The break-even point, a term frequently used in business, may be said to be the dollar volume of sales required in order that all costs may be met without giving rise to any loss or gain—*i.e.*, to enable a company to “break even” or to offset each dollar of expense with a dollar of revenue. It may be used to designate the percentage of the normal volume at which the company must operate in order to meet its expenses. The expenses which must be offset by sales revenues consist of the fixed expenses which will be incurred and the variable expenses applicable to the volume at the break-even point. A concern with a relatively low percentage of fixed, as compared with variable, expenses, will have a lower break-even point than a concern whose fixed expenses are higher in proportion. Thus, if fixed expenses are to variable expenses as 2 is to 7, the break-even point will be lower than it would be if fixed expenses and variable expenses are as 4 is to 5.

To find the break-even point, we divide the total of the fixed

expenses by the sales margin per dollar of sales over the variable expenses per dollar of sales. This is shown in the following illustration:

Sales		\$600,000	100%
Less: Variable cost of sales			
Materials and labor	\$240,000		
Manufacturing expenses	65,000		
Selling and administrative expenses	<u>109,000</u>	414,000	69%
Margin to cover fixed costs		<u>\$186,000</u>	<u>31%</u>
Less: Fixed costs			
Manufacturing expenses	\$120,000		
Selling and administrative expenses	<u>30,000</u>	150,000	25%
Profit from operations		<u>\$ 36,000</u>	<u>6%</u>
Add: Non-operating profit such as interest and dividends received		12,000	2%
Total profit		<u>\$ 48,000</u>	<u>8%</u>

Out of every dollar of sales, in the case just considered, 69 cents (or 69%) is needed to take care of the variable costs per unit. A balance of 31% is available to cover the additional costs which must be met by the company. In this case, \$12,000 of non-operating income is available to offset the same number of dollars of fixed expenses. Deducting this amount from \$150,000, we have a balance of \$138,000 of expenses which must be met by sales revenues. Since each sales dollar will contribute 31 cents toward these expenses, the total sales dollars to be secured in order to meet the net fixed charges can be obtained by dividing \$0.31 into \$138,000. The product, \$445,161, is known as the break-even point and represents the sales volume to be obtained if losses are to be avoided. If we regard the normal sales of \$600,000 as 100%, then we can secure the break-even percentage as follows:

$$\frac{74.193}{600,000} \text{ or } 74\frac{1}{4}\% \text{ approximately.}$$

To prove the accuracy of this sales volume, we can check over calculations as follows:

Sales		\$445,161	100%
Less: Variable expenses totaling 69%		<u>307,161</u>	<u>69%</u>
Margin available for fixed expenses		<u>\$138,000</u>	<u>31%</u>
Less: Fixed expenses	\$150,000		
Offset by other revenues of	<u>12,000</u>	<u>138,000</u>	<u>31%</u>
Net profit or loss		<u>—</u>	<u>—</u>

Break-even charts are often prepared. From these charts, the break-even point or percentage may be determined by approximation. A break-even chart prepared in conventional form is in Fig. 62. Lines plotted on the chart are based on the figures just presented.

In preparing the chart, we draw, parallel to the base or abscissa, a line representing the fixed costs. If fixed costs are constant at all capacities, then the line representing fixed costs will be parallel to the base regardless of the capacity at which the plant operates. The variable costs which fluctuate with the volume of output are superimposed on the fixed-cost line. A line is drawn to connect the sales-volume points at two capacities—zero (0) and one hundred per cent (100), which in this case represents normal. The point of intersection of the sales diagonal and the variable-cost diagonal superimposed on

the fixed-cost line represents the break-even point. The chart indicates that this is at about 75% of plant capacity. If the plant operates at less than this capacity, losses will arise as shown by the triangle on the chart. If the plant operates at greater than 75% capacity, then profits will be secured.

In the light of the previous discussion showing the relationship of volume changes, price changes, and managerial efficiency, we can see readily that the break-even point will move up and down the scale with each price change, volume adjustment, or increase or decrease in managerial efficiency. For example, an increase in unit price, if not offset by a shrinkage in volume, would cause the break-even point to move to the left and down the scale, thereby increasing the size of the upper (or "gain") triangle and decreasing the size of the lower (or "loss") triangle. An increase in volume would cause the same effect. On the other hand, any increase in fixed or variable costs brought about by managerial inefficiency or any other reason would also be unfavorable and cause the break-even point to move upward and to the right on the scale.

Slight modifications are occasionally made in the construction of the break-even chart. Costs or expenses are often divided into fixed, semi-variable, and variable costs. The semi-variable costs, or semi-fixed costs, as they are sometimes called, do not remain constant in amount at all capacities, nor do they increase proportionately. Rather, they tend to remain constant for certain capacities, then increase or step up to a higher figure at a higher level of capacity. Assume that a textile mill has five foremen in its weaving division. A degree of flexibility in organization exists which enables it to adjust volume without increasing or decreasing variable expenses proportionately. Five foremen will be needed whether volume remains stationary, decreases slightly, or increases slightly. An expense of this type is semi-variable.

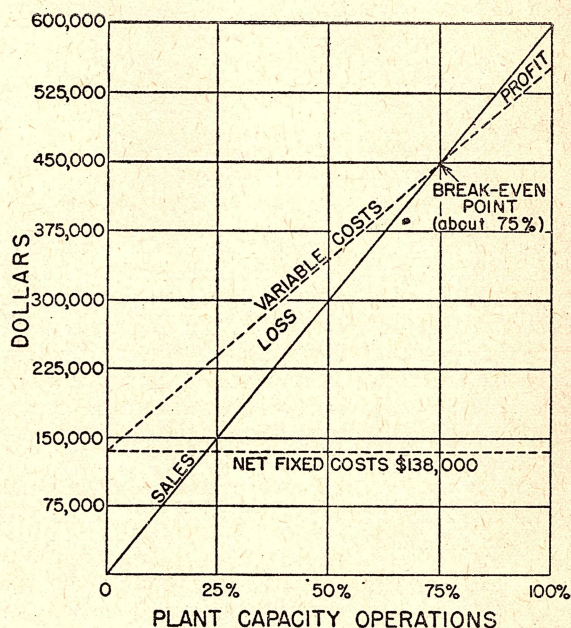


Fig. 62

For example, a plant may incur during normal operations total costs of \$100,000, of which \$24,000 may be fixed and the remainder variable and semi-variable, as indicated in the following table:

PERCENTAGE OF PLANT CAPACITY	FIXED COSTS	SEMI- VARIABLE	VARIABLE	TOTAL
0	\$24,000	\$ 500	\$ —	\$24,500
25	24,000	4,000	16,000	44,000
50	24,000	7,000	32,000	63,000
75	24,000	9,600	48,000	81,600
100	24,000	12,000	64,000	100,000

In preparing the chart, we find it immaterial whether fixed, semi-variable, or variable costs are represented by the lower line. The points should be so plotted that at any given capacity the total costs are properly represented on the chart. Often the sales line is represented as a third-degree curve. The sales price per unit would depend upon the volume available for sale. The smaller the volume, the greater the sales price which could be obtained per unit of product; as the volume increased, additional units would have to be sold at reduced prices. If each unit of product, regardless of volume, could be sold at the same price per unit the sales line on the break-even chart would always be a straight line.

TEST YOUR KNOWLEDGE OF BREAK-EVEN POINTS AND CHARTS

- 9 The Chemo Company is operating at normal capacity and sales amount to \$800,000 at its present rate of operations. Fixed costs total \$360,000, variable costs total \$400,000, and a profit of \$40,000 has been obtained. Due to a shortage of skilled labor, the company finds it necessary to curtail operations. Assuming no change in sales prices per unit, prepare a statement showing the sales volume required in order to break even. Prove your figures.
- 10 The Directors of the Chemo Company have analyzed the solution submitted to them in problem 9. They have come to the conclusion that they are justified in increasing the sales price per unit of product by 20%. It is not believed that this increase in price will result in any drop in volume. Prepare a break-even chart. At what capacity will the company have to operate, according to your solution, in order to break even?
- 11 The sales volume of the Genteel Company, manufacturers of nail polish remover, totals \$320,000. Fixed costs aggregate \$192,000, and variable costs total \$160,000, resulting in a loss of \$32,000. Prepare a statement showing (in terms of present capacity) the capacity at which the company must operate in order to break even. (*Hint: Add the loss to the fixed costs to determine the total amounts to be covered by the percentage of the sales dollar available for fixed expenses.*) Prove your answer.
- 12 Prepare a break-even chart for the Genteel Company. Assuming an increase of 20% in sales price without any corresponding increases in fixed or variable costs, or reduction in volume, determine the capacity at which the company would have to operate in order to break even on the basis of the proposed 20% increase in sales revenues.

EXECUTIVES' BONUSES

Many corporations have devised wage-incentive systems to spur employees to greater productivity. Many of these systems are very simple, but others are complicated. If the wage-incentive plan has been carefully prepared, the workmen usually understand the method of operation and are able to calculate the premium or bonus to which they are entitled as a result of increased effort and productivity. Foremen often share in these bonuses and as a result are desirous that the workmen perform efficiently and keep defective output at a minimum. This policy of offering incentives often extends to the top, or "key", executives. Many of them receive, in addition to their salaries, a percentage of the profits. Since profits cannot be determined until provision has been made for all expenses, the calculation of the bonus or premium to which an executive is entitled must be determined mathematically.

Illustrative Problem

Let us assume that the Lektrik Company has a contract with its sales manager whereby he is to receive, in addition to his salary, a bonus of 7% of the net profits after the deduction is made for federal income taxes. For the purpose of the problem, assume that the corporation must pay a federal income tax of 40% of net profits. The federal income tax is not deductible in arriving at the net profits on which the tax is payable. If the net profit before deducting either the bonus or the federal income taxes totals \$125,000.00, how much is due the sales manager, as a bonus?

Let X = bonus to be paid

and F = federal income tax.

Then $X = 7\% (\$125,000.00 - X - F)$,

$F = 40\% (\$125,000.00 - X)$.

a
b

Substituting for F in a, we have

$$X = 7\% [(\$125,000.00 - X) - 40\% (\$125,000.00 - X)] = \$5,038.39.$$

The sales manager would be entitled to receive a bonus of \$5,038.39

Proof

Net profit before bonus and taxes	\$125,000.00
Less: Bonus	5,038.39
Net profit on which federal income tax is payable	\$119,961.61
Less: Federal income tax @ 40%	47,984.65
Net profit on which bonus is payable	\$ 71,976.96
Bonus (@ 7%)	\$ 5,038.39

To carry the illustration one step further, let us assume that the assistant sales manager is also placed on a profit-sharing basis and that he is to receive 5% of the net profits after taxes, but before any deduction is made for his bonus. No change is to be made in the bonus percentage to be paid to the sales manager. For the purpose of the problem, assume a net profit before taxes and bonuses of \$125,000

and a federal income tax rate of 40%. (Bonuses are deductible in obtaining net profit subject to federal income taxes.)

Let X = bonus of sales manager,
 Y = bonus of assistant sales manager,
 F = federal income tax.

Then $X = 7\% (\$125,000.00 - X - Y - F)$ a
 $Y = 5\% (\$125,000.00 - X - F)$ b
 $F = 40\% (\$125,000.00 - X - Y).$ c

Substituting for F in a and b, we have

$X = 7\% [\$125,000.00 - X - Y - 40\% (\$125,000.00 - X - Y)],$ d
 $Y = 5\% [\$125,000.00 - X - 40\% (\$125,000.00 - X - Y)].$

Clearing each of the above, first for X :

$X = 7\% (\$125,000.00 - X - Y - \$50,000.00 + 0.40X + 0.40Y)$

$X = \$5,250.00 - 0.042X - 0.042Y$

$1.042X + 0.042Y = \$5,250.00$ e

Clearing for Y :

$Y = 5\% (\$125,000.00 - X - \$50,000.00 + 0.40X + 0.40Y)$

$= \$3,750.00 - 0.03X + 0.02Y$

$0.98Y + 0.03X = \$3,750.00$ f

Clearing equations e and f of decimals,

$Y = \$3,676.83 \quad X = \$4,890.19 \quad F = \$46,573.19$

	<i>Proof</i>		
	SALES MANAGER'S BONUS	ASSISTANT SALES MANAGER'S BONUS	FEDERAL TAX
Net income before deductions	\$125,000.00	\$125,000.00	\$125,000.00
<i>Less:</i>			
Sales manager's bonus	\$ 4,890.19	\$ 4,890.19	\$ 4,890.19
Assistant sales manager's bonus	3,676.83	—	3,676.83
Federal income tax	46,573.19	46,573.19	—
Total deductions	<u>\$ 55,140.21</u>	<u>\$ 51,463.38</u>	<u>\$ 8,567.02</u>
Net income	\$ 69,859.79	\$ 73,536.62	\$116,432.98
Sales manager's 7% bonus	\$ 4,890.19		
Assistant sales manager's 5% bonus		\$ 3,676.83	
Federal tax (40%)			\$ 46,573.19

TEST YOUR KNOWLEDGE OF EXECUTIVES' BONUSES

- 13 The president of The Iredel Company has a contract with the company which entitles him to a salary of \$12,000 per year and a bonus of 5% of the net profits remaining after the payment of income taxes. If the net income of the company before bonus and taxes totals \$250,000 and the federal tax rate is 40%, what is the amount of the president's bonus?
- 14 The president of The Iredel Company has an option clause in his contract which permits him to take 8% of the net profits before the deduction for income tax. Under the option, his salary is to be regarded as an advance made on account of his bonus. Would the president be wise in exercising the option? Indicate the amount he would lose or gain by exercising the option.
- 15 The president and the vice president of The Jewel Company are entitled to bonuses of 8% and 5%, respectively, of net profits before any deductions for income taxes. The bonuses are to be deducted for the purpose of

ascertaining net profits available for bonus computations. To what amount is each officer entitled if the profits before bonuses and taxes totaled \$100,000?

CONSOLIDATED STATEMENTS

Corporation officials often employ excess working capital in acquiring the stocks of companies in which they would like to exercise some measure of control. A policy looking toward ultimate control of the activities of other corporations is often followed, and shares of stock are acquired at market prices. If over 50% of the stock of another company is obtained, the company making the investment may consolidate the assets and liabilities of the company in which it has secured a dominant interest with its own assets and liabilities. Like assets of both companies are added and the same procedure is followed for the liabilities. The investment in the stock of the company known as the *subsidiary* is carried as an asset upon the books of the company which acquired the stock as an investment. The latter company is known as the *parent company* in the consolidation. It is evident that, if the assets and liabilities of both companies are to be combined, the stock ownership of the parent company in the subsidiary must be offset against the capital stock and surplus of the subsidiary. Assume that Company X acquired 80% of the stock of Company Y at the time when Company Y's stock was worth \$120 per share according to its books. Immediately upon the acquisition of this stock, the balance sheets of each company showed the following condition:

Company X

ASSETS		LIABILITIES AND CAPITAL	
Fixed assets	\$100,000	Fixed liabilities	\$ 20,000
Current assets	100,000	Current liabilities	30,000
Investment in Company Y (800 shares)	96,000	Capital stock, 2,000 shares (par, \$100 per share)	200,000
		Surplus	46,000
Total assets	<u>\$296,000</u>	Total liabilities and capital	<u>\$296,000</u>

Company Y

ASSETS		LIABILITIES AND CAPITAL	
Fixed assets	\$ 80,000	Fixed liabilities	\$ 50,000
Current assets	130,000	Current liabilities	40,000
		Capital stock, 1,000 shares (par, \$100 per share)	100,000
		Surplus	20,000
Total assets	<u>\$210,000</u>	Total liabilities and capital	<u>\$210,000</u>

In combining the balance sheets of both companies, we cancel out the investment of \$96,000 in the stock of Y upon the books of X against

\$80,000 of the capital stock of Y and \$16,000 of the surplus of Y. The consolidated balance sheet is prepared as follows:

ASSETS	X	Y	ELIMI- NATIONS	CONSOLIDATED BALANCE SHEET
Fixed assets	\$100,000	\$ 80,000		\$180,000
Current assets	100,000	130,000		230,000
Investment in Y	96,000		\$ 96,000*	
Total	<u>\$296,000</u>	<u>\$210,000</u>	<u>\$ 96,000</u>	<u>\$410,000</u>
LIABILITIES				
Fixed liabilities	\$ 20,000	\$ 50,000		\$ 70,000
Current liabilities	30,000	40,000		70,000
Capital stock	200,000	100,000	\$ 80,000*	220,000†
Surplus	46,000	20,000	16,000*	50,000†
Total	<u>\$296,000</u>	<u>\$210,000</u>	<u>\$ 96,000</u>	<u>\$410,000</u>

The preparation of a consolidated balance sheet in practice is much more difficult than the one shown in the above illustration. It is customary to indicate the value of the capital stock and the share of the surplus applicable to the minority interest.

If the company which is controlled owns some of the stock of the controlling company, the apportionment of the minority interest in the surplus and earnings is exceedingly complicated.

Illustrative Problem

Companies A, B, and C are affiliated companies. An examination of the books and records shows the following ownership:

Company A owns 40% of the stock of B, and 60% of the stock of C; Company B owns 20% of the stock of A, and 30% of the stock of C; Company C owns 10% of the stock of A, 40% of the stock of B, and 10% of the stock of C.

Company A controls Company C through the ownership of 60% of its stock. This enables it to control Company B, since it owns 40% of Company B stock and can vote the 40% of the stock of Company B which is owned by Company C. Twenty per cent (20%) of B stock is held by minority shareholders. The ten per cent (10%) of C's stock not owned by Companies A and B had been issued and was subsequently re-acquired by Company C in settlement of a debt. It is now held as an asset by C. Stocks purchased are carried upon the books of each company at original cost to them.

The assets of each company are shown on page 1084, as of December 31, 1943. Assume that no liabilities exist.

* The item of \$96,000 eliminated from the assets is offset by \$80,000 of capital stock of Y and \$16,000 of the surplus of Y. Company X paid \$96,000 for an 80% interest in Y and we must eliminate 80% of the capital stock of Y and 80% of the surplus of Y.

† The capital stock and surplus extended to the consolidated balance sheet columns may be analyzed as follows:

Capital stock:		
Majority interest	\$200,000	
Minority interest in Y	<u>20,000</u>	\$220,000
Surplus:		
Majority interest	\$ 46,000	
Minority interest in Y	<u>4,000</u>	50,000
Total capital and surplus		<u>\$270,000</u>

	A	B	C
Investment in stock of A	—	\$ 10,000	\$ 6,000
Investment in stock of B	\$ 20,000	—	24,000
Investment in stock of C	25,000	15,000	5,000
Other assets	105,000	125,000	115,000
Total assets	<u>\$150,000</u>	<u>\$150,000</u>	<u>\$150,000</u>
Capital* (consisting of capital stock and surplus)	<u>\$150,000</u>	<u>\$150,000</u>	<u>\$150,000</u>

Required: A statement allocating the assets of each company to the shareholders of the stock including the minority shareholders of Company B.

Solution

The 10% of Company C stock owned by Company C should be subtracted from the 100% of its stock. Company A owns 60% of the 90% outstanding (or $\frac{2}{3}$), and Company B owns 30% (or $\frac{1}{3}$).

Let X =net worth of Company A,
 Y =net worth of Company B,
 Z =net worth of Company C.

Then $X = \$105,000.00 + 40\% Y + \frac{2}{3}Z$ a

$Y = \$125,000.00 + 20\% X + \frac{1}{3}Z$ b

$Z = \$115,000.00 + 10\% X + 40\% Y$ c

$15X = \$1,575,000.00 + 6Y + 10Z$ d

$15Y = \$1,875,000.00 + 3X + 5Z$ e

$10Z = \$1,150,000.00 + X + 4Y$ f

Adding d and f, $14X - 10Y = \$2,725,000.00$ g

Multiplying e by 2 and subtracting d from the result,
 $-21X + 36Y = \$2,175,000.00$ h

Solving for X and Y in g and h, by adding $\frac{1}{2}$ of equation g to g and adding to h

$Y = \$298,214.28$ $X = \$407,653.06$

Substituting values for X and Y in c and solving for Z ,
 $Z = \$275,051.02$.

	NET WORTH OF EACH COMPANY	PERCENTAGE OF STOCK OWNED BY OUTSIDERS	VALUE OF NET ASSETS ASSIGNABLE TO OUTSIDE SHAREHOLDERS
A	\$407,653.06	70%	\$285,357.14
B	298,214.28	20%	59,642.86
C	275,051.02	None	None
	Total of assets		<u>\$345,000.00</u>

* It has been demonstrated that the capital or net worth of a business enterprise is equal to the difference between the assets and liabilities. The equation used was
assets - liabilities = capital.
If no liabilities are owing, then the equation will read
assets = capital.
In the illustration, the capital of each company will be the same as the assets of each company.

REVISED BALANCE SHEETS AS OF DECEMBER 31, 1943

(After giving effect to revision of values of securities owned)

	A	B	C
Investment in stock of A	—	\$ 81,530.61	\$ 40,765.31
Investment in stock of B	\$119,285.71	—	119,285.71
Investment in stock of C	183,367.35	91,683.67	—
Other assets	105,000.00	125,000.00	115,000.00
Total assets	\$407,653.06	\$298,214.28	\$275,051.02
Net worth	\$407,653.06	\$298,214.28	\$275,051.02

TEST YOUR KNOWLEDGE OF CONSOLIDATED STATEMENTS

- 16 The Conso Corporation owns a subsidiary through 100% stock ownership. The assets and liabilities of each company are shown below:

	CONSO CORPORATION	SUBSIDIARY
Sundry assets	\$150,000	\$100,000
Invested in stock of subsidiary	75,000	—
	<u>\$225,000</u>	<u>\$100,000</u>
Sundry liabilities	\$ 15,000	\$ 25,000
Capital stock	200,000	75,000
Surplus	10,000	—
	<u>\$225,000</u>	<u>\$100,000</u>

- In preparing a consolidated balance sheet, what items will you eliminate?
- 17 Assume that \$75,000 represented an 80% interest in capital and surplus of the subsidiary at the time of purchase. If the Conso Corporation paid exact book value, what amount of surplus must have appeared upon the books of the subsidiary at the time the 80% interest was purchased?
- 18 Assume the same facts as shown in problem 17. What value should be placed on the minority interest?
- 19 Assume the same facts as in problem 17. What value should be placed on the majority interest in surplus?
- 20 The Boards of Directors of the Weld Metal Company and its affiliates, the Alloy Metal Company and the Bronze Weld Company, have asked their accountants to submit a statement showing the net worth of each company after giving effect to revisions in the value of securities of each affiliate owned by the others. The Weld Metal Company owns 70% of the stock of the Alloy Metal Company and 40% of the stock of the Bronze Weld Company. The Alloy Company owns 30% of the stock of the Weld Metal Company and 30% of the stock of the Bronze Weld Company. The Bronze Weld Company owns 20% of the stock of the Weld Metal Company and 30% of the stock of the Alloy Metal Company. Other assets of each company are as follows:

Bronze Weld Company	\$100,000
Alloy Metal Company	125,000
Weld Metal Company	150,000

There are no outstanding liabilities in any of the companies. Prepare a statement showing the net worth of each company. The 30% of the stock of the Bronze Weld Company not owned by Alloy or Weld Metal represents a minority interest, and the 50% of the Weld Metal stock not owned by the Bronze Weld Company or the Alloy Metal Company represents the controlling interest.

- 21 Prepare a statement showing the division of the assets of the companies in problem 20 among the outside shareholders.

**INTERDEPENDENT STATE
AND FEDERAL TAXES**

Among the many taxes paid by a corporation are taxes levied on net income. Many corporations find

that, in addition to paying federal income taxes, it is necessary to pay State income taxes. In determining the amount of income which is subject to the State income tax, a corporation under the laws of some of the States may deduct the federal income tax but not the State tax to be paid. In other cases, the State law allows the corporation to deduct not only the federal tax but also the State income tax before calculating the State tax. The State income tax is *always allowed* as a deduction in determining the net income subject to the federal income tax. The deductions to be made before determining income subject to taxes are shown in the following summary in those cases in which State and federal taxes are deductible before determining the State tax.

	INCOME SUBJECT TO	
	FEDERAL TAXES	STATE TAXES
Net income before deduction of income taxes	\$xx,xxx	\$xx,xxx
Less:		
State tax on income	xx	xx
Federal tax on income	—	xx
Income subject to federal income tax	xxx	
Income subject to State income tax		xx

The interdependent relationship of State and federal income taxes is evident from the above summary. The federal income tax cannot be determined until the State income tax, which is deductible on the federal return, is known. However, the State income tax cannot be obtained unless the net income subject to the tax is known. In calculating this income, the federal and State tax is deductible. The amount of each tax can be secured through the use of an algebraic formula.

Illustrative Problem

Assume that a corporation is subject to a federal income tax of 40% and a State income tax of 5%. The net income before deducting any of these taxes is \$100,000. What is the amount of the federal tax which must be paid? What is the amount of the tax to be paid to the State?

Let

$X = \text{federal tax}$

$Y = \text{State tax.}$

Then

$X = 0.40 (\$100,000.00 - Y)$

$Y = 0.05 (\$100,000.00 - X - Y)$

$X = \$40,000.00 - 0.40Y$

$0.40Y = \$40,000.00 - X$

Multiplying c by 2.5,

$Y = \$100,000.00 - 2.5X$

Substituting in b,

$X = \$38,834.95 \text{ (federal tax)}$

Substituting for X in d,

$Y = \$2,912.62 \text{ (State tax)}$

a

b

c

d

e

Proof

FEDERAL TAX RETURN		STATE TAX RETURN	
Net income before deducting		Net income before deducting	
State tax	\$100,000.00	taxes	\$100,000.00
Less: State tax	<u>2,912.62</u>	Less: Federal tax	\$38,834.95
Net income subject		State tax	<u>2,912.62</u>
to federal tax	\$ 97,087.38		41,747.57
Federal tax at 40%	<u>\$ 38,834.95</u>	Net income subject	
		to State tax	\$ 58,252.43
		State tax at 5%	<u>\$ 2,912.62</u>

These calculations are illustrative only and would apply only to a corporation subject to a federal income tax of 40% and a State income tax of 5%, provided both taxes were deductible on the State return.

If the income subject to the State income tax is calculated in a different manner than the income subject to the federal income tax, adjustments of the formulae will have to be made. If a corporation is subject to other federal taxes, such as the excess profits tax and the declared value excess profits tax, the State and federal taxes may be determined by the "trial and error" method, or by more elaborate formulae. Accountants have prepared tables of composite rates to be used in those cases in which corporations are subject to numerous federal taxes.

TEST YOUR KNOWLEDGE OF INTERDEPENDENT STATE AND FEDERAL TAXES

- 22 Assume that a corporation has a net income of \$80,000 before deducting federal and State taxes. The federal rate is 40% and the State rate is 6%. The State tax is deductible on the federal return before calculating the federal tax. The federal and State taxes are deductible on the State return for the purpose of the State tax. What is the amount of the State tax to be paid?
- 23 Assume that the State tax is deductible on both the federal and State return, and that the federal tax is not deductible for purposes of the State return. What is the amount of the federal income tax for the corporation mentioned in problem 22?
- 24 Assume that \$30,000 of the \$80,000 income of the corporation mentioned in problems 22 and 23 was earned in an adjacent State and is not subject to the State tax rate of 6%. The federal and State taxes are deductible in determining the net income subject to State tax. The State tax is deductible in determining the net income subject to the federal tax. Calculate the federal tax and the State tax, assuming rates of 40% and 6%, respectively.
- 25 The president of the Logan Company has a contract with it under which he receives 10% of the net profits as a bonus. The company is subject to a State income tax of 5% and a federal tax of 40%. The State tax and the bonus are deductible in determining profits subject to federal taxes. The bonus, the State tax, and the federal tax are deductible for purposes of the State tax. If profits before deducting these totaled \$200,000, determine the federal tax. Prove your answer.

Solutions to Problems and Exercises

Answers achieved by readers may differ by a cent or two in some instances if different tables or different methods of calculation are employed. Such discrepancies, since they are minor, should not be a cause of concern.

MATHEMATICAL BASIS OF ACCOUNTING

1 \$11,000	2 \$15,000	3 \$45,000	4 \$5,828,000
5 \$5,200	6 \$2,969	7 \$11,955	
8 \$279,802	9 \$1,311,000	10 \$12,000	
11 \$17,000	12 J., \$19,500; G., \$14,500		
13 (a) \$155,000; (b) \$135,000	14 Prof., \$5,000		
15 \$8,000	16 \$54,000	17 Loss, \$2,000	
18 (a) \$64,500 (b) \$239,500			
19	CASH BASIS	ACCRUAL BASIS	
Profit	\$3,870	\$4,487	
Net worth	\$9,120	\$9,737	
20 \$2,021	23 \$40.34, \$107.73	24 Radio A	
25 \$3.00	26 \$22.50	27 \$13.56	
28 \$12.47	29 \$20.19	30 \$39.57	
31 \$39.03	32 \$12.00	33 \$2.37	
34 \$13.77	35 \$571.51	36 \$563.68	
37 (a) \$64.91 (b) \$63.70 (c) \$64.02			
38 2.5 yr.	39 3.5%	40 18 yr., 2 mo., $5\frac{1}{2}$ d.	
41 \$16.96	42 Gain, \$16.63	43 \$5,022.50	
44 \$1,529.06	45 (a) \$51.61 (b) No		
46 \$603.38	47 Yes	48 A, \$5,850; B, \$4,410	
49 \$1,485.95	50 20 yr.	51 6%	
52 \$2,654.57	53 \$11,925.69	54 \$5,485.57	
55 \$50,001.50	56 (a) \$404.00 (b) \$824.32		
57 \$14,100	58 \$10,500	59 \$54,800	
60 \$450	61 \$3,000		
62 N.W., \$37,775; N.P., \$3,975			
63 \$3,990.93	64 \$160.20		
65 (a) \$1,250,455.14 (b) \$1,245,423.53 (c) \$49,968.39			
66 \$90,199.78	67 (a) \$372.05 (b) \$2,872.05		
68 4.5014%	69 \$155,265.70	70 \$2,833.39	
71 (a) \$1,888.93 (b) \$1,943.40			
72 (a) \$944.46 (b) \$889.99			
73 \$110.01	74 \$11,854.90	75 \$4,858.49	
76 Amount, \$1,012.50; rate, 20.25%			
77 \$1,962.50	78 (a) \$1,150 (b) \$95.83 (c) \$5,675		
79 (a) \$8,000 (b) Loss, \$3,000 (c) \$500			
80 Loss, \$235	81 (a) \$4,000 (b) \$8,000	82 7.6%	
83 (a) \$1,500 (b) \$735 (c) \$360.15 (d) \$840.35			
84 (a) \$938.04 (b) \$503.32 (c) \$322.12			
85 25 yr.	86 11.25%	87 28.6%	
88 \$19,995.50	89 (a) Truck 2 (b) \$45.20		
90 $86 : D_1 = \$414.59$; $87 : \$626.69$; $89(b) : \$53.83$			
91 $D_1 = 71.554$; $D_2 = 75.13$			
92 $86 : D_1 = \$99.8822$; $D_2 = \$103.8775$; $87 : D_1 = \$435.6566$; $D_2 = \$453.0828$; $89(b) : \$38.80$			
93 (a) \$340 (b) \$290	94 5		
95 \$491.07	96 <i>Illus.</i> : \$939.73; 95 : \$548.56		
97 \$2,378.57; \$1,889.61; \$2,470.45; \$1,983.62			
98 \$66,811.42	99 \$4,005,757.46		
100 \$11,110,188.11			

ANALYSIS AND INTERPRETATION

1 \$1,237,000	2 C.A.	93.3	C.L.	26.7
	F.A.	3.1	F.L.	67.1
	D.C.	1.5	N.W.	6.2
	O.A.	2.1		
		100.0		100.0

3	No significant change			
4	Cost of goods sold 92.48	Other income 0.02		
	Other op. expen. 1.06	Other expen. 0.09		
	Dividends 0.41			
5	Loss, 1%	6	Dividends, \$7089.30	
			Balance to surplus, \$16,069.08	
7	(a) 0.0368	(b) 2.26	(c) 0.262	8 Latter
9	(a) 50%	(b) 20%	(c) 150%	(d) 800%
			(e) $66\frac{2}{3}\%$	
10	Reduce from 200% to $166\frac{2}{3}\%$			
11	(a) 62.8	(b) 342.4%	12	(a) 58.06
13	56.63		(b) 55	(c) 55
14		'40	'41	'42
	J	110.1	146.2	172.5
	K	128.4	113.5	134.3
	L	126.4	145.4	138.0
15		'39	'40	'42
	J	68.5	75.3	118.0
	K	88.1	113.1	118.3
	L	69.1	87.3	95.3
16		'41	'42	'43
	Cash	90.8	93.1	85.3
	Securities	100.7	124.1	76.6
	Receivables	106.7	77.5	89.1
	Inventories	86.2	118.1	153.4
17	C.A. : C.L.	200.5	N.E. : N.S.	9.6
	(C.A. - I.) : C.L.	86.7	Div. : N.S.	8.1
	C.A. : T.A.	73.5	N.S. : R.	411.9
	N.W. : F.A.	180.3	N.S. : I.	254.9
	N.W. : T.L.	84.5	N.S. : F.A.	420.9
	O.E. : N.S.	90.4	N.S. : N.W.	233.3
18	97.7		19	103.5
20	(a) Application (f) Source			
	Others—neither			

SPECIALIZED PROBLEMS

1 to 4 Net decrease, \$6,000			
(In computations, 000's omitted)			
1 $(8+2) - (1.2+2.8) = 6$			
2 $(27\frac{1}{3} + 7\frac{1}{3} + 1\frac{1}{3}) - 30 = 6$			
3 $(17+3.2+0.3) - 14.5 = 6$			
4 $(5+4) - (2+1) = 6$			
5 No; decr. net prof., \$11,600			
6 Yes; incr. net prof., \$30,500			
7 (a) Profit, \$72,200 (b) Profit, \$75,800			
8 40,988 units	9 \$720,000		
10 64 to 65% (Sales volume required, \$514,285)			
11 140% (Sales volume, \$448,000)			
12 85 to 86%	13 \$7,281.55		
14 Gain, \$125.85			
15 Pres., \$7,079.65; V.P., \$4,424.78			
16 Investment; capital stock			
17 \$18,750	18 \$15,000	19 \$10,000	
20 W.M., \$553,505.53; A.M., \$389,298.89; B.W., \$327,490.77			
21 W.M., 50% \$276,752.77			
B.W., 30% 98,247.23			
Total	\$375,000.00		
22 \$2,779.92	23 \$30,188.68		
24 Fed., \$31,583.01; State, \$1,042.47			
25 \$73,394.50			

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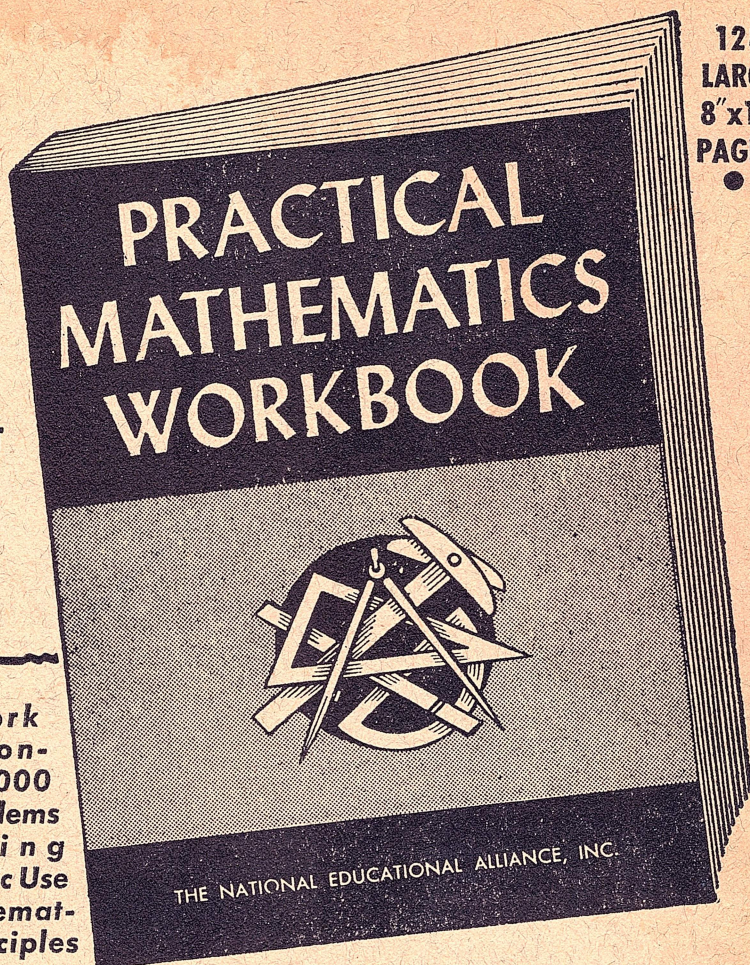
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